GRE - (Graduate Record Examination)

It is conducted by ETS - Educational Testing Services

Sections	Number of Questions	Alloted Time
◦ Verbal Reasoning (2 Sections) (130 - 170) marks	20 Question per Section Total Que 40	30 minutes per Sections Total time (60) mint
◦ Quantitative Reasoning (2 sections) (130 - 170) marks	20 Question per Section Total Que 40	35 minutes per Sections Total Time (70) mint
◦ Analytical Writing (Essay)	One "Analyse an Issue" Task and one "Analyse an Argument" Task.	35 minutes per Section Total time (70) mint

11/07/2018

Types of Questions

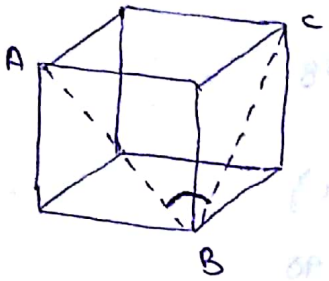
I. Numeric Ext (4Q)

Ex: Any two side of Δle are Equal Then it is ____.

Ans: Equilateral.

II Single answer choice (16Q)

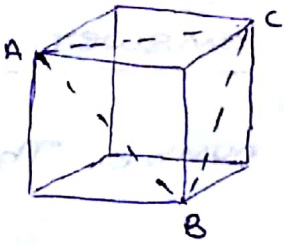
Ex:



Find ∠ABC ?

A) 45 B) 60 C) 90 D) 120 E) Cannot be determine

Answer: B.



ΔABC is equilateral Triangle

so ∠ABC = 60

III Quantitative Comparision (16Q)

Example 1:

Quantity A	Quantity B
30^{40}	70^{90}

A: Quantity A > Quantity B

B: Quantity B > Quantity A

C: Quantity A = Quantity B

D: Cannot be determine

Ans: B.

Example 2: Quantity A 100^{100} Quantity B $99^{99} + 99^{98}$

A: Quant A > Quant. B

B: Quant B > Quant. A

C: Quant A = Quant B

d: Cannot be determine

Ans: A

100^{100} , $99^{99} + 99^{98}$

100^{100} , $99^{98}(99+1)$

100×100^{99} , 100×99^{98} //

(iv) One or More Than one answer choice - (4Q)

Ex: What are the prime number That will divide A.

Where $A = 2^{31} + 2^{32} + 2^{33} + 2^{34}$

A) 2 , B) 3 C) 5 d) 7 E) 11

Ans: A, B, C

$2^{31}(1+2+4+8)$

$2^{31+1}(15) = 2^{30}$ //

$2 \times 3 \times 5$

Number system

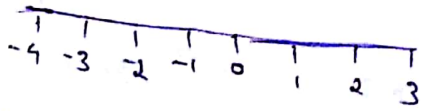
Representation of Numbers

* Natural no set (N) :-

  ~~N~~ = {1, 2, 3 ... ∞}

* Whole number set (W) :- {0, N}

* Integer no. set (Z) :- {-ve number, W}



* Rational no. set (Q) :- {Decimal, Z}

  $Q = \left\{ \frac{p}{q} : q \neq 0 \right\}$

* Irrational no. set (S) : $\{\sqrt[n]{a}\}$

* Complex no. set (C) : {a ± ib}

  (R) Real no

  Imaginary (I)

∴ R - {N, W, Z, Q, S}

Ex: Super set of all the no's?

A) N B) R C) Z D) Q E) None

Ans: E)

complex no doesn't comes under real number (R)

Super set

Ex: Skin is super set of hair

Classification of Numbers

Even - $\frac{n}{2}$: R_0 ; odd - $n/2$: R_1

Even + Even = Even	E × E = E	$(Even)^n$ = Even
E + odd = odd	E × O = E	$(odd)^n$ = odd
O + E = O	O × E = E	
O + O = E	O × O = O	∀ n ∈ N
O + O + O = O		

→ Prime no- Number of divisor = 2

1 to 100 - 25 prime ; 1 to 50 - 15 prime ; 50 to 100 - 10 prime

50 to 100 - {53, 59, 61, 67, 71, 73, 79, 83, 89, 97}

→ Composite no:- No of divisiors > 2

Example

Finding a prime number

~~231 ≈ (15)²~~ =

> Que: 231

Sol: find closest prime number - 15
have have square near to 231

$(15)^2 = 225$

$231 = (15)^2 \Rightarrow \{2, 3, 5, 7, 11, 13\}$ | use prime no

check individually with, 2, 3, 5, 7, 11, 13 | 2, 3, 5, 7, 11, 13

2 - x , 3 - ✓

$\frac{231}{3} = 77$ //

> Que: 131

$\Rightarrow (11)^2 = 121$

now {2, 3, 5, 7}

× × × ×

131 is a prime number.

Ex:

Any twenty prime no, added results to Even, which of the following cannot be prime no. in it.?

A) 31 B) 21 c) 11 d) 7 E) 2

Ans: E

twenty (prime) = Even

twenty (odd's) = Even

> Factors (divisiors)

Numbers = $a^m \times b^n \times c^p$ $\{a, b, c \in \text{prime no.}\}$

- No of factors = $(m+1)\times(n+1)\times(p+1)$
- No of prime factors = $m+n+p$
- No of different prime factor = No of bases

Ex: 18

Ans: 18 can be written as

$18 = 2^1 \times 3^2$

So, No. of factors = $(1+1)\times(2+1) = 6$

No. of prime factors = $1+2 = 3$

No. of different prime factor = 2

Ex: 720 and also find odd and even factor

Sol: $720 = 2^4 \times 3^2 \times 5^1$

No of factors = $5\times3\times1 = 30$

No of prime factors = $4+2+1 = 7$

No of different prime factor = 3 - {2, 3, 5}

No of odd factors = $(2+1)\times(1+1) = 6$

No of even factors = (No of factors - No of odd factors)

$= 30-6 = 24$

Ex: 21600

Sol: $21600 = 2^5 \times 3^3 \times 5^2$

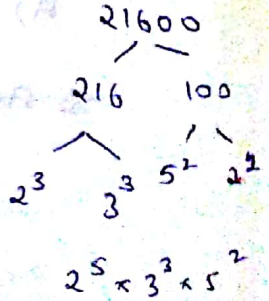
No of factors = $6\times4\times3 = 72$

No of prime factors = $5+3+2 = 10$

No of different Prime fact = 3

No of odd factor = $4\times3 = 12$

No of even factor = $72-12 = 60$



Ex: how many perfect sq. factors exist for 21600

Sol: $21600 = 2^5 \times 3^3 \times 5^2$

perfect sq. contain power 2 or even. so

$2^5 = 2^2 \times 2^2 \times 2 \times 2^0 = 3$ — we can write it $(2^0 = 1)$

$3^3 = 3^2 \times 3^1 \times 3^0 = 2$

$5^2 = 5^2 \times 3^0 = 2$

So now total perfect sq. factors

$3 \times 2 \times 2 = 12$ //

Unit place value

Thousand Tens Hundredth

3789.652 → Thousanth

Hundred unit Tenth

Ex: find units place of

1) $3^3 = 27 = 7$

2) $8^4 = \quad = 6$

3) $3^{379} = \ldots = ?$

Note

units pattern length of 2, 3, 7, 8 is (4) (have 4-results) mean

" " " " 4, 9 is (2) (have 2-results)

" " " " 1, 5, 6 is (1) (have 1-result)

→ Do now for 3^{379} u-patter length = 4

$4\overline{)397}$ (36, 37, 36, 7)

$4\overline{)379}$ 94 (36, 19, 16, 3) (Remainder = 3)

now $3^{(\text{remainder})} = 3^3 = 27 = 7$ //

(27: 1 unit place)

unit place of $3^{379} = 7$ //

Note

$(\text{Base})^{\text{Power}} \Rightarrow$ $\overline{)\text{Power}}$ (Units Pattern length of the base)

$\frac{\text{Power}}{\text{R}=}$

If $R \neq 0 \Rightarrow$ units place value of $(\text{Base})^{\text{Power}} = (\text{Base})^{R}$

If $R = 0 \Rightarrow$ units place value of $(\text{Base})^{\text{Power}} = (\text{Base})^{\text{unit pattern length of base}}$

Que ⇒ unit place value of 3^{379} of A (find A)

Sol: $A = 33^{33} + 97^{97} + 55^{55}$ 10

$= 3^{33} + 7^{97} + 5^{55}$

$= 3^1 + 7^1 + 5^1$

$= 12 + 3 = 15 \Rightarrow 5$ //

Que: unit's place value of

$379^{379} \times 477^{477} \times 576^{576} \times 393^{393}$

$9^{379} \times 7^{477} \times 6^{576} \times 3^{393}$

$9^1 \times 7^1 \times 6 \times 3$

63 18

24

4 is unit place value

| 1 1
| 379
| 377
| 758
| 4
| 35

*** Que what is unit place value of $(43^{43} - 53^{53})$

Sol here 43^{43} is less than 53^{53}

$43^{43} - 53^{53}$

$3^{43} - 3^{53}$

$3^3 - 3^1$ $(27 - 3)$

$\Rightarrow 7 - 3 = ?$ (here 43^{43} is less Than 53^{53})

$= 4$ (how borrow one as we do for $\Rightarrow 103 - 76 =$

$\Rightarrow 10 - 4$ $\Rightarrow 103^{13}$

$\Rightarrow \boxed{6}$ // $\frac{76}{27}$ $(13-6)$ (like That

Que: what is the unit's place value of

Sol: $(131)^{131} \times (132)^{132} \times (133)^{133} \times \ldots (139)^{139}$

$= 0$ (because 5 is There in This series)

Que $171^{171} \times 173^{173} \times 175^{175} \times 177^{177} \times 179^{179}$ 11

Sol here unit place value is 5 (because 5 is present)

Remainder concept

> Dividend = Divisors × Quotent + Remainder

$37 = 7 \times 5 + 2$

$\frac{30}{7} = R_2$, $\frac{20}{7} = R_6$, $\boxed{\frac{10}{11} = R_{10}}$, $\boxed{\frac{2}{7} = R_2}$

Que now $\frac{600}{7} = ?$

Sol $\frac{20 \times 30}{7} = R_6 \times R_2$

$= R_{12} \simeq R_5$ $\left(7\overline{)12}\,(1\right.$ $\left.\frac{7}{5}\right.$

So, $\frac{600}{7} = R_5$

Que $\frac{10}{7} = 3$

Ans $= \frac{30-20}{7} = R_2 - R_6$

$= R_{-4+7}$

$= R_3$

Que $\frac{50}{7}$

Ans: $\frac{30+20}{7}$

$= R_2 + R_6$

$= R_8 \simeq R_1$

$\rightarrow$

$\frac{XYZ}{10} = R_Z$

$\frac{379}{10} = R_9$

$\frac{3764217}{10} = R_7$

$\frac{XY(Z)}{5}$ — $Z<5 \rightarrow R_Z$; $Z=5 \rightarrow R_0$; $Z>5 \rightarrow R_{Z-5}$

eg: $\frac{379}{5} = R_{9-5} = R_4$

eg: $\frac{372142}{5} = R_2$

$\frac{77775}{5} = R_0$

Que) what is the remainder for $\frac{372^{377}}{5}$

Ans) $\frac{372^{377}}{5} \Rightarrow R_2$ (suppose $\frac{372^1}{5} = R_2$)

Que) $\frac{8^{11}}{10} = \frac{8^3}{10} = \frac{8\times 8\times 2}{10} = R_2$

$\frac{8^{15}}{10} = \frac{8^3}{10} = \frac{8\times 8\times 2}{10} = R_2$

Que) $\frac{2^{3006}}{7}$

$= \frac{2\times 2\times 2\times 2\times 2\times \cdots \times 2 \text{ (3006 times)}}{7}$

$= \frac{8\times 8\times 8\times \cdots \text{ (1002 times)}}{7}$

$= R_1 \times R_1 \times R_1 \cdots \times R_1$ (1002 times)

$= R_1$

Que) $\frac{3^{379}}{7}$

$= \frac{3\times 3\cdots \text{ (378 times)} \times 3}{7}$

$= \frac{729\times 729\cdots \text{ (63 times)} \times 3}{7}$

$= \frac{R_1\times R_1\times \cdots \text{ (63 times)} \times 3}{1}$

$= R_3$ //

$6\overline{)379}$ (63, 36, 19, 18, 1)

H.W

$= \frac{9^{3794}}{7} \times \frac{8^{3765}}{9}$

$= \frac{9^{3794}}{7} \times \frac{8^{3765}}{9}$

$= \frac{729 \times 729 \times \cdots (1264) \times 9^2}{7} \times \frac{64 \times 64 \times 64 \times (1882) \times 8}{9}$

$= R_1 \times R_1 \times R_1 \times \cdots \underline{R_4} * R_1 \times R_1 \times R_1 \cdots \underline{R_8}$

$= R_4 * R_8$

$= R_{32}$

R_4 , R_8

13

$\frac{9^1}{7} = 2$

$\frac{9^2}{7} = 4$

$\frac{9^3}{7} = 1$

$\frac{9^4}{7} = 2$

Rule :-1 Prime/co-prime

LCM (no.s) = product of the nos

HCF (no's) = 1

Example : LCM ⇒ (5,7) = 35

HCF ⇒ (5,7) = 1

LCM (2,3,5) = 30 | LCM (9,8) = 72

HCF (2,3,5) = 1 | HCF (9,8) = 1

Note : co-prime : when two number don't have ~~similarity~~ any common number ~~or not~~ eg: (2,3), (5,7), (22,17)

Rule :-2 Smallest no. is a factor of all other no's recussively.

LCM (no's) = Biggest no.

HCF (no's) = Smallest no.

Example

LCM (4, 8, 64) ⇒ LCM = 64

HCF (4, 8, 64) ⇒ 4

Rule :-3 (valid only on two no's)

LCM (no's) = common numbers × product of co-primes

HCF (no's) = common multiples among all

Example: LCM = (18, 24)

= 6(3,4)

LCM = 6×3×4 = 72

HCF = 6

LCM = (4, 8, 60, 96)

= (4, 8, 96, 60)

= (96, 60)

= 12(8, 5)

= 12×40

LCM = 480

HCF = 4

⇒ LCM & HCF on Decimals

0.36, 1.2, 1.8

here hundredth is the highest Decimal place so multiply it by 100

LCM = (0.36, 1.2, 1.8) × 100

= (36, 120, 180)

= (120, 180)

= 60 (2, 3) = 60 × 2 × 3 = 360

LCM = (0.36, 1.2, 1.8) = $\frac{360}{100}$ = 3.6

HCM = (0.36, 1.2, 1.8) = $\frac{12}{100}$ = 0.12

⇒ LCM & HCF on ~~Dec~~ Fractions

LCM (fraction) = $\frac{\text{LCM (Numerators)}}{\text{HCF (Denominators)}}$

HCF (fraction) = $\frac{\text{HCF (Numerators)}}{\text{LCM (Denominators)}}$

• LCM $\left(\frac{1}{3}, \frac{2}{7}, \frac{4}{5}\right)$

$\frac{\text{LCM}(1, 2, 4)}{\text{HCF}(3, 7, 5)}$ = $\frac{4}{1}$ = 4 //

•

$\frac{P}{q} = \frac{Nr}{Dr} = \frac{\text{Dividend}}{\text{Divisor}}$

⇒ Divisors – LCM

⇒ What is the ~~best~~ least possible no. that is divisible by 8, 10 and 12?

Sol

$\frac{n}{8}, \frac{n}{10}, \frac{n}{12}$

$n = LCM(8, 10, 12) = 120$

HCF (miss)

17/07/18

17

FRACTION

Fraction $\rightleftharpoons$ Decimal

Decimal

Non-Recurring (or) Terminating Decimals

Ex: $\frac{1}{4} = 0.25$

Ex $\frac{3602}{100} = 36.02$ — 3.624

$\frac{1801}{50}$

Recurring or Repeating or Non-Terminating Decimals

Pure Repeating

Ex: $\frac{1}{3} = 0.333..\infty$

$= 0.\overline{3}$

$32,\overline{45}, 0.1\overline{25}$

Mixed Repeating

Ex: $\frac{1}{6} = 0.1666..\infty$

$= 0.1\overline{6}$

Que: What is the sum of Numinator and denominator of the fraction which results to $0.24\overline{6}$

Sol:

$$0.24\overline{6} = \frac{246-24}{900}$$

{ no. after point (right) − non bar number (right) / 9 * (no. of zero equal to the non-bar no.) }

$$= \frac{222}{900} \quad \cancel{450} \; 150 \quad \cancel{\text{II}} \; 37$$

$$= \frac{37}{150}$$

∴ from question Num + deno = 37 + 150 = 187

➢ Non-Repeating Decimal → fraction

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

Pure Repeating Decimals → Fraction

Ex: $0.\overline{3} = \frac{3}{9} = \frac{1}{3}$, $4.\overline{2} = 4\frac{+2}{\times 9} = \frac{38}{9}$

$0.\overline{24} = \frac{24}{99} = \frac{8}{33}$

MIXED Repeating Decimals → Fraction

$0.12\overline{3} = \frac{123-12}{900} = \frac{111}{900} = \frac{37}{300}$

→ BODMAS

$16+14\div 2$, $(16+14)\div 2$

$16+7=23$, $30\div 2=15$

This method is applicable to Addition and subtract

Ex: $\frac{5}{6}+\frac{3}{4} = \frac{5}{6}\nearrow\searrow\frac{3}{4} = \frac{(5\times 4)+(6\times 3)}{24} = \frac{38}{24} = \frac{19}{12}$

Multiplication

$\frac{3}{7}\times\frac{5}{4} = \frac{15}{28}$

Division

$\frac{\frac{3}{7}}{\frac{5}{4}} = \frac{3}{7}\times\frac{4}{5} = \frac{12}{35}$

Comparition

→ which is greater

~~$\frac{11}{12}$ ~~ $\frac{8}{9}$ $\frac{11}{12}$, $\frac{8}{9}$

$\frac{11}{12} \therefore \frac{8}{9}$

11×9, 12×8

99, 98

So

$\frac{11}{12} > \frac{9}{8}$

$\frac{13}{15}$, $\frac{8}{9}$

13×9, 15×8

117, 120

Fraction = $\frac{\text{His contribution}}{\text{Total capacity}}$, Total capacity = Product of the denominators

Model-1

Que: Jhon spent $\frac{1}{3}$ of his salary towards rent, $\frac{3}{4}^{th}$ of remaining salary spent toward expenditures, $\frac{2}{3}$ of remaining salary spent towards taxes, which ~~frac~~ factors of his salary left for Then?

Sol: multiply all denominator value = $3 \times 4 \times 3$

$= 36$

Rent = $\frac{1}{3} \times 36 = 12$, $36 - 12 = 24$

Expenditures = $\frac{3}{4} \times 24 = 18$, $24 - 18 = 6$

Taxes = $\frac{2}{3} \times 6 = 4$, $6 - 2 = 2$

fraction = $\frac{2}{36} = \frac{1}{18}$

Model-2

In a zoo, $\frac{1}{7}$ of living creatures are birds, $\frac{1}{5}$ of living creature are aquatic animals, $\frac{3}{4}$ of remaining are wild animals if 690 animals are reptiles Then, how many living creatures are There in a zoo?

Sol: Total = $7 \times 5 \times 4 = 140$ (all denominator)

Birds = $140 \times \frac{1}{7} = 20$

aquatic = $\frac{1}{5} \times 140 = 28$

wild = $\frac{3}{4} \times 92^{23} = 69$

$140 - 48 = 92$

$140 - 117 = 23$

Total	reptiles
140	23
x	690

$690 \times 140 = x \times 23$

$x = 4200$

Model-3 (Grid model)

~~1/2 of students =~~

In a class 1/2 of students are girls, ~~and~~ 3/4 of girls are Junior and 2/3 of students are Juniors.

what fraction of Juniors boys exist in the college?

Sol

Total students -

1/2 of stu - Girls ⇒ 1/2 × 24 = 12

2/3 of students - Juniors ⇒ 2/3 × 24 = 16

3/4 of girls are Juniors ⇒ 3/4 × 12 = 9

	J	S	Total
B	16-9 = 7	8-3 = 5	24-12 = 12
G	9	12-9 = 3	12
Total	16	24-16 = 8	24

Total student = 2 × 3 × 4

= 24

PERCENTAGE

18/07/2018

$$\text{Percent (\%)} = \frac{\text{His contribution}}{\text{Total capacity}} \times 100\%$$

Que: What percent of three digit numbers Exist in which all the digits are same?

Sol * No of items = [(max) - (min)] + 1

$$\frac{9}{900} \times 100 = 1\%.$$

* 20% of 60,000 = 12000
  40% of 120 = 48
  30% of 16 = 48
  0.2% of 12 = 0.024
  0.1% of 0.1% of 0.001
  = 0.000000001

{first multiply two number then do cancelation of two zero above and down}

⇒ INCREMENT / DECREMENT

$$\uparrow\downarrow\ \% = \frac{\text{Difference}}{\text{Intial Value}} \times 100\%.$$

(A) 120 → 160

$$= \frac{160-120}{120} \times 100$$

$$= 33.33\% \uparrow$$

(B) 160 → 120

$$= \frac{120-160}{160} \times 100$$

$$= 25\% \downarrow$$

Note = 1

Percentage Increased from x to y is not same as percentage decreased from y to x.

Que Mr Jones joined in a new job his salary is increased by 50%. later his salary is decreased by 30%. because of his irregularity what percent more salary is he getting compare to the original?

Sol let consider - 100 so now

50% increase = 150%.

30% decrease = ~~70%~~

now

150% of 100 = 150

70% of 150 = 105

compare to the orignal = ~~100~~ $\frac{105-100}{100} \times 100$

= 5%.

Ans

Short cut

$$x + y + \frac{x \cdot y}{100} = 50 - 30 + \frac{50 \times (-30)}{100}$$

$$= 20 - 15 = 5\%$$

Note :-

When percentages sperated on difficult total we cannot add/subtract the % age directly are

$$x + y + \frac{x \cdot y}{100}$$

Fundament add/subtract

A: 60% of T

B: 40% of T

(60-40)% of T = 500

$\frac{20}{100} \times T = 500$

T = 25,000

x + 5000 + x = 25000

x = 10,000 //

Note

When percentages operated on same total we can add/subtract the given percentages directly.

Note: When given information in the Question is only in percentages, Then we can assume total as 100.

Example: 40% of family are having P.C in 1998, 50% of family were increased in 2012 and no of family having PC increased to 80% what percent of family are having PC in 2012?

Sol

	family	P C
1998	100	40
2012	150	72

40% of 100 = 40

180% of 40 = 72 → (increased from yr 1998 to 2012) which (100+80)% = 180

(%age of family having P.C/242) $\frac{72}{150} \times 100 = \frac{720}{15} = 48\%$

(%age of family increased having PC in 2012) $= \frac{72-40}{150} \times 100 = 21.33\% \uparrow$

One of The 500 business PP

leave page.

Profit & loss

$P = S.P - C.P$

$L = C.P - S.P$

$d = M.P - S.P$

$P\% = \frac{P}{C.P} \times 100$

$L\% = \frac{L}{C.P} \times 100$

$D\% = \frac{d}{M.P} \times 100$

$S.P = \frac{100+P}{100} \times C.P$

$S.P = \frac{100-L}{100} \times C.P$

$S.P = \frac{100-D}{100} \times M.P$

S.P = Selling price , M.P = Mark price , D = Discount
C.P = cost Price , L.P. Least price , P = Profit
L = loss.

Short cuts

⇒ CP = 4000 , P = 20% , SP = ?

S.P = (100+20)% = 120% of C.P

S.P = 120% of 4000 = 4800

P = 20% of 4000 = 800

⇒ CP = 5000; L = 30% SP = ?

SP = (100-30)% = 70% of CP

SP = 70% of 5000 = 3500

L = 30% of 5000 = 1500 //

and also L = C.P - S.P = 5000 - 3500
= 1500 //

⇒ MP = 8000; D = 20% } S.P = ?

S.P = (100-20)% of 8000
= 80% of 8000 = 6400.

D = M.P - S.P = 8000 - 6400 = 1600

⇒ MP = 12,000 , D = 20% , P = 20%.

Profit = $ ——— ?

Sol:

S.P = 80% of 12000 = 9600 ⇒ S.P = 9600

S.P = 120% of C.P

9600 = 1.2 of CP ⇒ C.P = 8000

P = SP - C.P = 9600 - 8000 = 1600

⇒ MP → 60% more than C.P , D = 20%.

Profit = ?

Sol

Let CP = 100 , M.P = 160

S.P = 80% of 160

= 128

Profit = S.P - CP

= 28 % //

Shortcut

$x + y + \frac{x.y}{100}$

$= 60 - 20 + \frac{60 \times 20}{100}$

$= 40 + \frac{(-1200)}{100}$

= 28 //

(for discount and loss use -ve symbole before number)

⇒ M.P = 10,000, D_1 = 20% ; D_2 = 30% , D = ?

SP = (100-20)% of (100-30)% of 10,000

= 80% of 70% of 10,000

$= \frac{56}{100} \times 10,000 = 5600$

$\frac{80 \times 70}{100 \times 100} = \frac{5600}{100 \times 100} = 56$

There are Two discounts so -ve symbole mean we have to subtract

Two percentage are here so denominater should have 2 hundred.

D = M.P - S.P = 10,000 - 5600

D = 4400

⇒ S.P = 120$ (Two of Each cycle) ; P = 20% ; L = 20%

now use formula

$x + y + \frac{x \cdot y}{100} = 20 - 20 + \frac{20 \times (-20)}{100}$

$= 0 - \frac{400}{100} = -4 = 4\%$ loss

Note

-ve symbolise loss, +ve symbolise profit.

= find single discount when two discounts are provided.

eg: $D_1 = 30\%$ $D_2 = 10\%$

Sol: $= -30 - 10 + \frac{(-30 \times -10)}{100}$

$= -40 + \frac{300}{100}$

$= -37$

So single discount is 37%

⇒

Cycle - I

$SP_1 = 120$; $P = 20\%$.

$SP_1 = 120\%$ of C.P

$120 = 1.2 \times CP$

$C.P_1 = \frac{120}{1.2} = 100$

Cycle - II

$SP_2 = 120$; $L = 20\%$.

$SP_2 = 80\%$ of CP_2

$120 = \frac{80}{100} \times CP_2$

$CP_2 = 150$

here we have to find out Total loss or profit % eg and as we seen above both cycle have same 'S.P'.

Total CP : $CP_1 + CP_2 = 100 + 150 = 250$

SP : $SP_1 + SP_2 = 120 + 120 = 240$

$L\% = \frac{L}{CP} \times 100 = \frac{10}{250} \times 100 = \frac{100}{25} = 4\%$

Shortcut

⇒ CP of 10 pens = SP of 12 pens { as we seen it is loss so }

$L = \frac{10-12}{12} \times 100 = \frac{-2}{12} \times 100 = 16.66\%$. (loss)

formula

C.P of x-articles = SP of Y articles

$$P \text{ or } L = \frac{X-Y}{Y} \times 100$$

shortcut

$$\frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\%$$

$$= \frac{200}{1000 - 200} \times 100$$

$$= \frac{200}{800} = 25\%$$

Que: If The MP of an article is 5/4 times of cost Price and gives a discount of 10% on overall transaction how much profit or loss, it results to

Sol $MP = \frac{5}{4} \times C.P$, $SP = 100 - 10 = 90\%$.

SIMPLE INTEREST & COMPOUND INTEREST

$SI = \frac{PTR}{100}$

$CI = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

$A = P\left(1+\frac{RT}{100}\right)$

$A = P\left(1+\frac{R}{100}\right)^n$

$A = P\left(1+\frac{R_1+R_2\cdots+R_n}{100}\right)$

$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}+\cdots+\frac{R_n}{100}\right)$

P = Principel, ~~I = Interest~~, R = Rate, A = amount
T = Time., SI = Simple interest, CI = compound interest
A = amount, I = Interest

Relation b/t CI & SI is given as

$A = P + I \begin{cases} SI \\ CI \end{cases}$

S.I

M-1 ⇒ P = 300, R = 20% Pa, T = 2 years

for two years rate is 2×20 = 40%.

So 100+40 = 140%.

A = 140% of P = 140% of 3000 = 4200

(or) I = A − P = 4200 − 3000 = 1200

I = 40% of 3000 = 1200.

M-2 ⇒ P = 3000 R_1 = 20% R_2 = 30%.

A = 150% of 3000 = 4500

I = A − P = 4500 − 3000 = 1500

→ If P = 3000 R = 20% Pa, after one years he repays 1600 and clears the loans in next two years what is the amount he pay at the end?

Sol

I_1 = 20% of 3000 = 600

so = 3000 + 600

= 3600

1600 he paid so = −1600

2000

A = 140% of 2000 = 2800

CI = compound Interest

M-1 ⇒ P = 3000 R = 20% compounded annually, T = 2yrs

A = 120% of 120% of 3000 = 4320 (for two yrs 120% × 120%.)

CI = (A−P) = 4320 − 3000 = 1320

⇒ P = 3000 R_1 = 20%, R_2 = 30% compounded annually

A = 120% of 130% of 3000 = 4680

CI = A − P = 4680 − 3000 = 1680

M-2 ⇒ $CI_{9^{th}\ Years}$ = 20,000 R = 30%, $CI_{10^{th}\ Year}$ = ?

$CI_{10^{th}}$ = 20,000 + 30% of 20,000

$CI_{10^{th}}$ = 20,000 + 6,000 = 26,000

formula :-

$$CI_{(n+1)^{th}\ Year} = CI_{n^{th}\ Year} + R\%\ of\ CI_{n^{th}\ Year}$$

⇒ P = 3000 R = 20% compounded $CI_{3^{rd}\ year} = ?$

$CI_1 = 20\%$ of 3000

$CI_1 = 600$

$CI_2 = 20\%$ of 3000 + 20% of 600

$CI_2 = 600 + 120$

$= 720$

$CI_3 = 20\%$ of 3000 + 20% of 1320

$= 600 + 204$

$CI_3 = 804$

for 2-yr CI = 600 + 720 = 1320

for 3-yr CI = 600 + 720 + 804 = 2184.

SI — 600 600 600 (here in SI the amount never change year by year)

CI — 600 720 864 (in CI the ~~interest~~ interest get added in the amount to get final year amount)

⇒ for 2yr

$CI - SI = Pr^2 \quad \left(r = \frac{R}{100}\right)$

$CI - SI = \frac{1}{2} r$ of SI

⇒ for 3yr

$CI - SI = Pr^2(3 + r)$

$CI - SI = \frac{1}{3} r(3 + r)$ of S.I

eg: CI - SI = 120, T = 2yrs, R = 20% pa

P = ?

$CI - SI = Pr^2$

$120 = P\left(\frac{20}{100}\right)^2$

$120 = P \times \left(\frac{1}{25}\right)$

$P = 120 \times 25 = 3000$

P = 3000

CI

Times	Year
3	2
27	? (3-2)

For How to find out

Times	Years
3 x	n
x^a	a-n

In SI

$A = 3P$ $\quad A = 27P$

Rate

$$P\left(1+\frac{RT}{100}\right) = A$$

$$P\left(1+\frac{R\times 2}{100}\right) = 3P$$

$$1+\frac{R}{50} = 3$$

$$\boxed{R = 100}$$

Time

$$P\left(1+\frac{100T}{100}\right) = 27P$$

$$1 + T = 27$$

$$\boxed{T = 26}$$

Averages

$\bar{x}$ = Sum of all items / Total no of items

$\bar{x} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}$ (combined average)

$\bar{w} = \frac{w_1 n_1 + w_2 n_2}{n_1 + n_2}$ (weight average)

eg: $\{2, 3, 7, 8\}$

$\bar{x} = \frac{20}{4} = 5$

→ if all no are same Then average is ~~same~~ also same

eg:- $\bar{x} = \frac{20}{4} = 5$

$\{5, 5, 5, 5\} \rightarrow \bar{x} = 5$

eg

→ $\{2, 3, 7, 8\}$ find average

Short cut

Sol: $\{2, 2, 2, 2\} \rightarrow \bar{x} = 2$

$+1, +5, +6$

$= \frac{12}{4} = 3$

$\bar{x} = 2 + 3$

$= 5$

{ first find out smallest number, make it common. Then see how much number is needed and add them and divide it with Total number

Qu) In a class room the average weight of 20 girls is 12kg and 10 boys is 14kg.

(a) What is the average weight of class

(b) The average of the class become '15' when we add a teacher to the class. what is the weight of the teacher

(c) 5-boys left the class what is the average of the 5-boys if average of the remaining student in the class is 12.5.

Sol)

a) $\{12, 12, 12, \ldots 12, 14, 14, 14 \ldots 14\}$

(20) (10)

$\{12 \ldots\ldots 12\} \rightarrow \bar{x} = 12$

$2 \times 10 = 20$

$= \frac{20}{30} \cdot 0.66$ → 0.66, 12.66

average weight of class = 12.66

b) $\{12, 12 \overset{(20)}{\ldots} 12, 14\,14 \overset{(10)}{\ldots} 14\}$

$\{15, 15 \overset{(20)}{\ldots} 15, 15 \overset{(10)}{\ldots} 15 \mid 15\} \rightarrow \bar{x} = 15$ ← ①-Teacher added.

$= (3 \times 20 + 1 \times 10)$

$= 70$

70 + 15 — (Teacher + class average) = 85

weight of the teacher = 85.

(c) $\{12 \overset{(20)}{\ldots} 12, 14 \overset{(5)}{\ldots} 14\}$

Given average 12.5

$\{12.5 \ldots\ldots 12.5, 12.5 \ldots 12.5\} \rightarrow 12.5$

$= +12.5 - 12$, $= 12.5 - 14$

$= 0.5$ $= -1.5$

$\Rightarrow 0.5 \times 20$, $\Rightarrow -1.5 \times 5$

10 , -7.5

$= 10 - 7.5$

$= 2.5$

2.5 is distributed in 5 boys those who left so

$\frac{2.5}{5} = 0.5$

$14 - 0.5 = 13.5$

or $(13.5, 13.5 \ldots, 13.5)$ (5)

Que) The average numbers of visitors in downtown library is 300 in a week days and 500 visitors on sundays, what is the average no. of visitors on Sunday in a particular month which starts with Sunday

no. of sundays when its starts with 1st day = 5 sunday
no of days = 30.5
= 25

Sol:

(5) (25)

{500... 500, 300, ... 300}

{300 ..... 300} → $\bar{x} = 300$

$200 \times 5 = \frac{1000}{30} = 33.33$ | $= \frac{33.33}{333.33}$ //

Que) ~~A = {3, 4, 6, 8, 10}~~

Que) Set A = {0, 4, 6, 8, 12} which of the items (numbers) in the set has to removed so that the average of set A, before removal ~~and after removed and~~ and after removal will be same.

Sol

$A = \{0, 4, 6, 8, 12\}$

$= \frac{0+12}{2} = 6$ //

$\frac{\text{First value} + \text{last value}}{2}$

Que) {320, 324, 328, ... 8000}

Sol $\bar{x} = \frac{320 + 8000}{2} = 4160$

→ Short cut / formula

*** If ∃ common difference between the items

Average = $\frac{F+L}{2}$

eg:- $\{x_1, x_2, x_3 \cdots x_4\}$ $\bar{x}_A = P$

$\{3x_1+2t, 3x_2+2t, \cdots 3x_n+2t\}$

$\bar{x} = 3P+2t$

→ $\{2,3,7,8\}$

$\bar{x} = \frac{20}{4} = 5$

×3 → $\{6,9,21,24\}$

$\bar{x} = \frac{60}{4} = 15$

$\bar{x} = 5 \times 3 = 15$

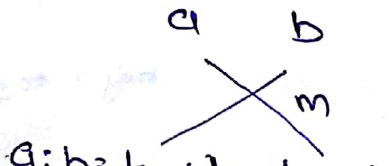
−1 → $\{1,2,6,7\}$ $\bar{x}$

$\bar{x} = 5-1 = 4.$ $[=5-1 = 4]$

Note

Averages holds all fundamental operators $(+, -, \times, \div)$

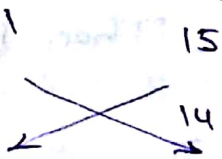
Aligation and mixture.



$a : b = |m-b| : |m-a|$

Ex:

Sol



$G : B = |14-15| : |14-11|$

$= |-1| : |3|$

$\boxed{G:B = 1:3}$

3- Boy

1- Girl

now $\{11, 15, 15, 15\}$

$\{11, 11 \ldots 11\}$ — $\bar{x} = 11$

$4 \times 3 = \frac{12}{4} = 3$

$\bar{x} = 11+3$

$\boxed{\bar{x} = 14}$

Ratio and Proportion

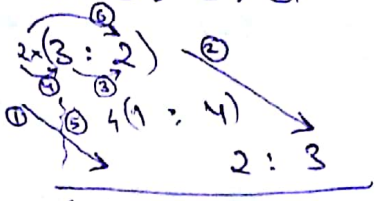
→ Ratio

x to y = $x:y = \frac{x}{y}$

Ex

$a:b = 3:2 \quad b:c = 1:4 \quad c:d = 2:3$

$a:b:c:d = ?$



$6 : 4 : 16 : 24$

1st
$3 \times 1 \times 2 = 6$
last
$2 \times 4 \times 3 = 24$
now
$3 \times 2 = 6$ (second no. is 2 so and 24 multiplied by 2 to get 2nd number)

$a:b:c:d = 3:2:8:12$

→ ~~Eitv~~ Red, Green, Blue.

$R = \frac{1}{3} \cdot G \quad B = \frac{1}{4} \cdot R$; Then find $G:R:B$

Sol

$R:G = 1:3, \quad B:R = 1:4$

as we see that the required sequence is not in order so for that we have to change it

$G:R = 3:1, \quad R:B = 4:1$

$G:R = 4(3:1)$

$R:B = 4:1$

$12 : 4 : 1$

$G:R:B = 12:4:1$

Share Value = $\frac{\text{It's contribution}}{\text{Among all}} \times$ Total capacity

Que 2(a) Given walnut and Almond in the ratio of 3:5 ~~and~~ in a packet, whose weight is 720 gms find the weight of almonds

Sol

$$\text{Almonds} = \frac{5}{3+5} \times 750 = 450 \text{ gms}$$

450 gms of almonds in 750 gms pack

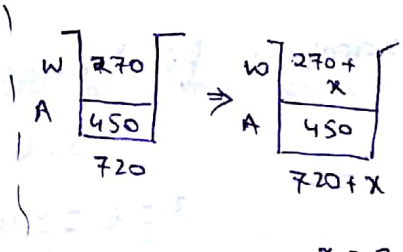
(b) If walnut and almond ratio is changed that is 5:3, Then what amount of more walnut is to be added.?

Sol

W : A = 5 : 3

$$\frac{270+x}{450} = \frac{5}{3}$$

150



$270 + x = 750$

$\boxed{x = 480 \text{ gm}}$

Que. if $A = \frac{1}{2} \cdot B$; $B = \frac{1}{2} \cdot C$ and total profit = 49,000

Then what is the profit share will 'B' gets?

Sol: Given A : B = 1 : 2 ; B : C = 1 : 2

A : B : C

1 : 2

1 : 2

1 : 2 : 4 ∴ A : B : C

(A + B + C = 4 + 2 + 1 = 7)

B's shares = $\frac{2}{7} \times 49{,}000 = 14{,}000$ /-

Proportions : The equality of two ratios is called Proportion.

$a:b::c:d$

here, $a:b = c:d$, we write $a:b::c:d$ and we say that a, b, c, d are in proportion

⇒ a and d are called extremes, while b and c are called mean terms.

⇒ To solve it we have to do product of b, c and it is equal to $a \times d$.

or

Product of means = Product of extremes.

Thus, $a:b::c:d = (b \times c) = (a \times d)$

$= a:b::c:d = a \times d = b \times c$

example: find missing number a

sol: 3, 6 and 10 ?

$3:6::10:?$

$?.3 = 10 \times 6$

$= 20$ //

Hence : $3:6::10:20$

⇒ $a:b::b:c$

$b^2 = ac$

$b = \sqrt{ac}$

Variations

Directly varying

~~If one sum~~ if first value increases, Then second value also increase, ~~vice versa~~.

$\downarrow\uparrow X \propto Y\uparrow\downarrow$ Then do cross multiplication

eg:- Petrol Travel

~~sol~~

20 — 500

? — 600

sol

$20^{4} \times 600 = ? \times 500$

24

24

Inversely varying

if first value increase/decrease, Then second value decrease/increase.

$\downarrow\uparrow X \propto \frac{1}{Y} \downarrow\uparrow$ Then do row multiplication.

eg: Mens ~~work~~ → hours

sol

20 — 8

? — 4

(here as men increases working hours decreases) ~~or same~~

for this do row Multi

$= 20 \times 8^{2} = ? \times 4$

$= 40$

Mr. Jhon Cena Travel with a speed of 60 mile/hr ~~and~~ ~~Too~~ time $4\frac{1}{2}$, with the same distance what is the speed if he ~~too~~ cover with in $2\frac{1}{2}$ hr.

Sol here if speed ↑ time ↓

so do row multi

~~60~~ Speed Time

60 — $4\frac{1}{2}$

? — $2\frac{1}{2}$

$? \times 2\frac{1}{2} = 60 \times 4\frac{1}{2}$

$? \times \frac{5}{2} = \overset{12}{\cancel{60}} \times \frac{9}{\cancel{2}}$

$= 108$ mile/hr

Que: A certain Automotive dealers sells only car and Trucks, and the ratio of cars to trucks on the lot is 1:3, if there are currently 51 trucks for Sales. how many car does dealer have for the sale.

Sol Given Cars and Trucks ratio 1:3 and currently There are 51 trucks for sale.

let x = cars

$1 : 3 = x : 51$

$3x = 51$

$\boxed{x = 17}$

17 - cars.

Time and Distance

⇒ Distance (D) = speed (S) x time (t); $t = \frac{D}{S}$; $S = \frac{D}{t}$

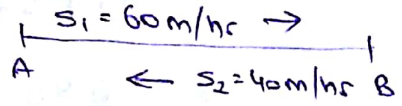
Time and distance contain.

1) Average speed.
2) Constant speed.
3) uniform speed
4) Relative speed.
5) usual speed
6) Train Problem.

⇒ Average speed = $\frac{\text{Total Distance}}{\text{Total time}}$

Ex: if speed of car moves from A to B and B to A is 60m/h and 40m/hr find its Average speed.

$A.S = \frac{\text{Total Dist}}{\text{Total time}}$ {let distance be 'x'}



$= \frac{2x}{\frac{x}{60} + \frac{x}{40}}$ {Time = $\frac{D}{S} = \frac{x}{60}, \frac{x}{40}$}

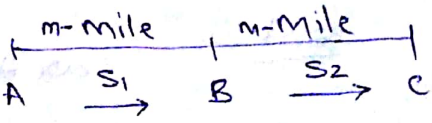
$= 40 \cdot \frac{2x}{\frac{100x}{2400}} =$

$A.S = 48$ //

Formulas.

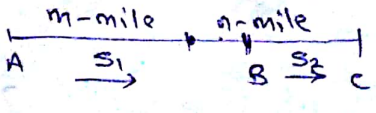
- if distance is same Then

$A.S = \frac{2 \cdot S_1 \cdot S_2}{S_1 + S_2}$



- if distance is not same Then

$A.S = \frac{(m+n) S_1 \cdot S_2}{m S_2 + n S_1}$



eg: if a car travels 20 miles with 20m/h, 60miles with 120m/h and 200mile with 200m/hr find A.S

Sol

let $m = 20m$, $n = 60m$, $p = 200m$

$S_1 = 20m/h$, $S_2 = 120m/h$, $S_3 = 200m/h$

$$A.S = \frac{Total\ Dist}{Total\ time}$$

$$= \frac{200 + 60 + 20}{\frac{20}{20} + \frac{60}{120} + \frac{200}{200}}$$

Time = $\frac{D}{S}$

$$= \frac{280}{1 + 0.5 + 1}$$

$$= \frac{280}{2.5} = 112\ m/h$$

⇒ Uniform speed.

eg: Two car covers same distance in $3\frac{1}{2}$hr and 5hr, is 320 and x find x.?

Sol

car A	320	$3\frac{1}{2}$
car B	?	5

{ if distance increases time also increase

$$? \times \frac{7}{2} = 320 \times 5$$

$$x = \frac{3200}{7} = 457.1\ miles.$$

⇒ A man traveled with $\frac{3}{7}^{th}$ of his usual speed, he is late to the party by 60-mint. How much time will he take, if he uses his usual speed?

Sol $\frac{3}{7}$ of his ~~used sp~~ usual speed, mean to travel with its original speed he need 4 more so+ That

$\frac{3+4}{7} = \frac{7}{7} = 1$ (1 means original speed)

Shortcut now,

Party	late / early
4	60'
(N)→ 3	?

$? \times 4 = 3 \times 60$ (15)

$= 45$

⇒ for normal calculation:-

let Speed = $\frac{3}{7}S$

time = $t+60$,

$D = S \times t$, $D = \frac{3}{7} \times S(t+60)$

$\cancel{S}t = \frac{3}{7} \times \cancel{S}(t+60)$

$7 \times t = 3(t+60)$

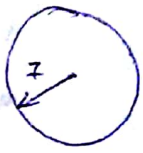
$7t = 3t + 180$

$180 = 4t$

$\boxed{t = 45}$

Constant speed.

Que A car is ~~travel~~ moving in a circular path whose radius is 7, with a speed of 60 m/h find how much further distance car ~~has~~ to move after 2hr of journey to reach its original (starting) point.



Sol

$D = 2 \times \pi \times r = 2 \times \frac{22}{7} \times 7$

$= 44$ //

$\boxed{D = 44}$

Speed = 60 m/h, time = 2hr

$D = s \times t$

$= 60 \times 2$

$= 120$

after 2hr =

44, 88, 132

$= 132 - 120$

$= 12$ //

Relative Speed

⇒ for car moving in same direction and opp direction

$(\text{same}_{\text{direc}})\ R.S = S_1 - S_2$; $(\text{opp}_{\text{direction}})\ RS = S_1 + S_2$

⇒ for boat moving in opp direction and same dir

$(\text{opp direct})\ R.S = B - W$; $(\text{same}_{\text{direction}})\ RS = B + W$

Train Problem.

Que. Train speed is 40mt/sec and bridge distance 1600 mts find time taken by train to cross bridge., take train length 400 mts

Sol

D = Actual distance + Train length

= 1600 + 400

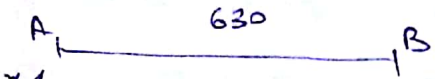
D = 2000

$$\text{Time } (T) = \frac{D}{S} = \frac{2000}{40} = 50 \text{ sec}$$

$$\boxed{T = 50 \text{ sec}}$$

Que Two car moves in opp. direction, distance b/t them is 630, Speed is 10m/h and 20m/h if after every hour it get double find at what distance both car going to meet from their origin.

Sol



M → ← N

10m/h	:	20
20	:	40
40	:	80

* M : N

1x : 2x

$3x = 630$

$\boxed{x = 210}$

'A' meets B at 210 miles from its origin

'B' " A " 420 miles " " "

630
210
420

Line, Angle, Polygon

→ **Line**

Point – • { which doesn't have any angle or area is zero.

Ray – O ⟶ A $\overrightarrow{OA}$ { continuously moving, or doesn't have any end.

line – A ⟷ B $\overleftrightarrow{AB}$ { It's two dimensional line

line segment – A X Y B $\overline{XY}$

Collinear points – A B C D

⇒

Intersecting line

Non-Intersecting (or) Parallel

Perpendicular lines.

Concurrent Lines

⇒ using n non-collinear points, no. of Maximum lines can be drawn

$$\frac{n(n-1)}{2}$$

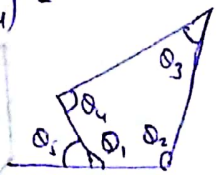
eg: if n=4

$$\frac{4(4-1)}{2} = 2 \times 3 = 6$$

Angle ∠

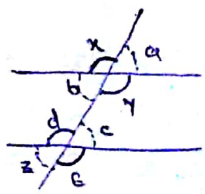
1. Acute Angle – $0 < \theta < 90^\circ$
2. Obtuse Angle – $90^\circ < \theta < 180^\circ$
3. Right Angle – $\theta = 90^\circ$
4. Straight Angle – $\theta = 180^\circ$
5. Reflexsive Angle – $180^\circ < \theta < 360^\circ$
6. Complete Angle – $\theta = 360^\circ$
7. Complementary Angle – $\theta_1 + \theta_2 = 90^\circ$
8. Supplementary Angle – $\theta_1 + \theta_2 = 180$
9. Linear pair Angle – sum of all angles = 180°
10. Congruent Angle – Equal angles
11. Central Angle – θ

12. Suspended Angle – $\angle x, \angle y, \angle z$
13. Opposite Angle – $(\angle\theta_1, \angle\theta_3)$ or $(\angle\theta_2, \angle\theta_4)$.
14. Adjacent Angle – $(\angle\theta_1, \angle\theta_1)$
15. Interior Angle – $\angle\theta_1, \angle\theta_2, \angle\theta_3, \angle\theta_4$
16. Exterior Angle – $\angle\theta_5$

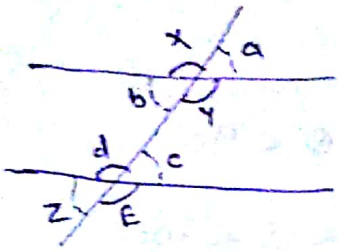


17. Vertically opp Angle – $\angle A = \angle b$, $\angle x = \angle y$

18. corresponding angle – ~~$\angle x = \angle y$~~ ; ~~$\angle a = \angle b$~~
$(\angle a, \angle c), (\angle b, \angle z)$
and also $\angle a = \angle$
$\angle b = \angle z$



19. Alternative interior Angle – $\angle b = \angle c$, $\angle y = \angle d$.



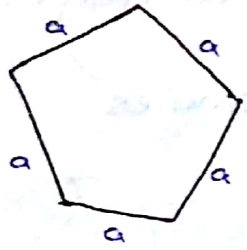
20. Alternative Exterior angle - $\angle a = \angle z$, $\angle x = \angle E$

21. Transversal interior angle - $(\angle y, \angle c)$, $(\angle b, \angle d)$
$\angle y + \angle c = \angle b + \angle c = 180$

22. Transversal Exterior angle - $(\angle x, \angle z)$, $(\angle a, \angle E)$
$\angle x + \angle z = \angle a + \angle E = 180$

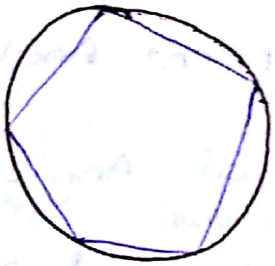
Polygons

Types of Polygons.

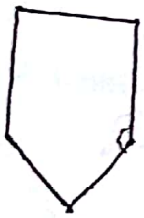


Regular

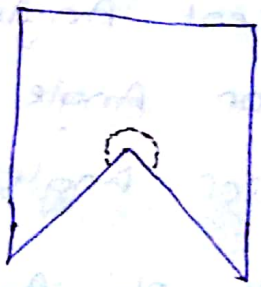
Irregular



cyclic Pentagon



convex Pentagon



concave Pentagon

→ Sum of all interior angles of a 'n' sided polygon
$= (n-2).180^\circ$

→ Sum of all exterior angles of n sided polygon
$= 360^\circ$

→ Each interior angles of n sided regular polygon
$= \frac{(n-2).180}{n}$

→ Each Exterior angles of n sided regular polygon
$= \frac{360}{n}$

→ No. of diagonals of n sided polygon $= \frac{n(n-3)}{2}$

26/07/2018

Triangles.

So

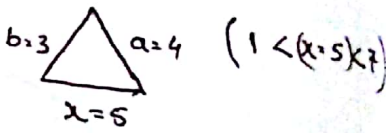
Triangles

Sides
- Equilateral Δ^{le}
- Isosceles Δ^{le}
- Scalar Δ^{le}

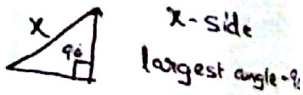
Angles
- Right angled Δ^{le}
- Acute Angled Δ^{le}
- Obtuse Angled Δ^{le}.
- Isosceles Right angle Δ^{le}

Properties

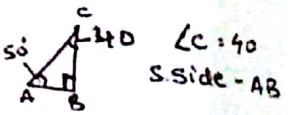
1. Sum of two sides > Third side.
2. If a, x, b are sides of Δ^{le} then
$$(a-b) < x < (a+b)$$



3. Largest side OPP, largest angle Exist



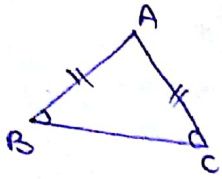
4. Smallest angle OPP, smallest side exist



5. Sum of all angles in a Δ^{le} = 180°

6. 

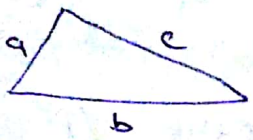
$\angle Z = \angle x + \angle Y$

7. 

$AB = AC$, $BC = AB$
$\angle B = \angle C$, $\angle C = \angle B$

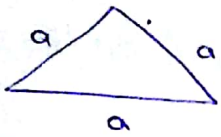
8. Area of an Δ^{le} = $\frac{1}{2}$ × base × height

Perimeter of an Δ^{le} = Sum of all side of Δ^{le}

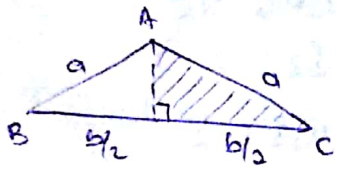
9. 

$A = \sqrt{s(s-a)(s-b)(s-c)}$

Semiperimeter $(s) = \frac{a+b+c}{2}$

10 

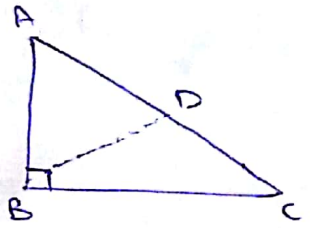
$A = \frac{\sqrt{3}}{4} \cdot a^2$; $P = 3a$, $H = \frac{\sqrt{3}}{2} \cdot a$.

11 

$h = \sqrt{a^2 - \frac{b^2}{4}} = \sqrt{\frac{4a^2-b^2}{4}}$

$A = \left(\frac{1}{2} \times \frac{b}{2} \times h\right) 2 = \frac{bh}{2}$

$A = \frac{b}{2}\left(\frac{\sqrt{4a^2-b^2}}{2}\right)$

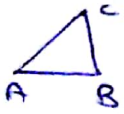
12 

$AB^2 + BC^2 = AC^2$

$\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

13. $AB^2 + BC^2 < AC^2$ - obtuse angled Δ^{le} - 

14. $AB^2 + BC^2 > AC^2$ - Acute Angled Δ^{le} - 

15. $AB^2 + BC^2 = AC^2$ - Right Angled Δ^{le} - 

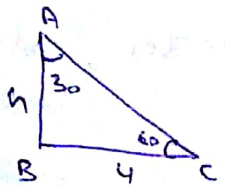
14. if $(3, 4, 5)$ - $3^2 + 4^2 = 5^2$

Then $2(6, 8, 10) - 6^2 + 8^2 = 10^2$

15. $30 : 60 : 90^\circ$; $45 : 45 : 90$; $1 : \sqrt{3} : 2$; $1 : 1 : \sqrt{2}$ } we got it from $\sin\theta, \cos\theta, \tan\theta$

eg: find the area of Δ^{le} ? where $\angle A = 30^\circ$
side $BC = 4$

Sol



$\angle A = 30$

$BC = 4$

$30 : 60 : 90$

$4 \times (1 : \sqrt{3} : 2)$

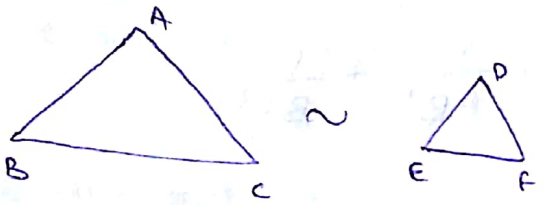
$4 : 4\sqrt{3} : 8$

Take 60° opposite side which is $4\sqrt{3} = h$

$$A = \frac{1}{2} \times 4 \times 4\sqrt{3}$$

$$\boxed{A = 8\sqrt{3}}$$

16.

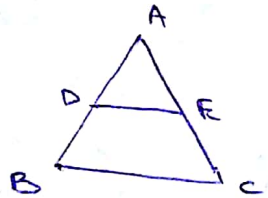


Then

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} = \frac{\text{Perimeter of } \Delta^{le} ABC}{\text{Perimeter of } \Delta^{le} DEF}$$

$$\frac{\text{Area of } \Delta^{le} ABC}{\text{Area of } \Delta^{le} DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

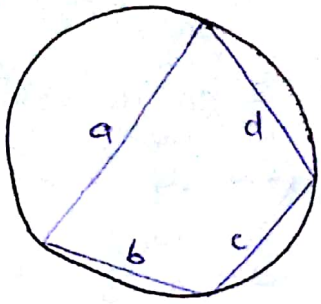
17.



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}.$$

Quadrilaterals

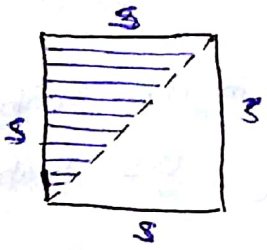
① Circlic Quadrilateral



$$\text{Area}(A) = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$s = \frac{a+b+c+d}{2}$$

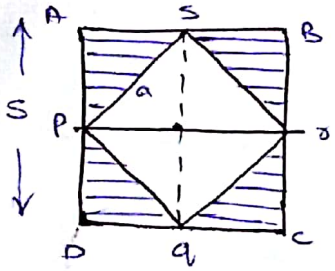
② Regular Quadrilateral



$A = S^2$

$P = 4S$

$D = \sqrt{2}.S$



Area of Pqrs

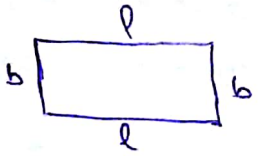
$= \frac{1}{2} \times$ Area of ABCD

$S = \sqrt{2}.a$

Area of shaded part = Area of un-shaded part.

③ Irregular Quadrilaterals.

Rectangle

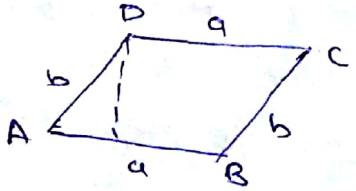


$A = l \times b$

$P = 2(l+b)$

$D = \sqrt{l^2+b^2}$

Parallelogram



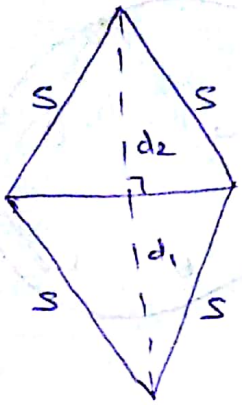
$A = b \times h$

$P = 2(a+b)$

$\angle A = \angle C, \quad \angle B = \angle D$

$\angle A + \angle B = \angle A + \angle D = 180$

Rhombus



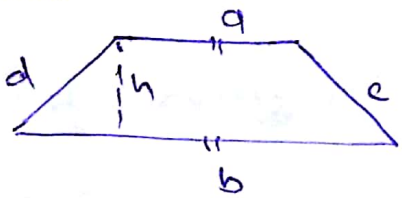
$A = \frac{1}{2} d_1 \cdot d_2$

$P = 4 \cdot s$

$s = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$

{ s = side length.

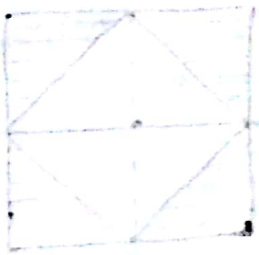
Trapezoid



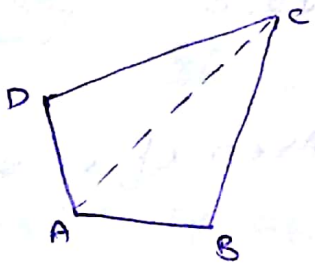
{ in UK - Trapezium
U.S - Trapezoid

$A = \frac{1}{2} h (a+b)$

$P = a+b+c+d.$



Trapezium

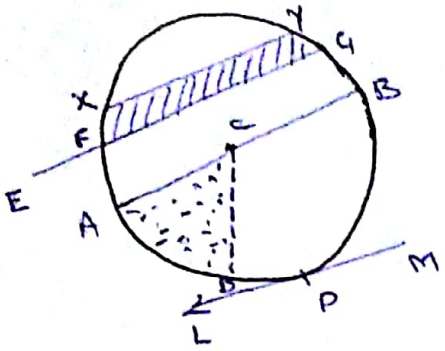


Area of ABCD
= Area of Δ^{le} ABC +
Area of Δ^{le} ADC

Circles

27/07/18

56

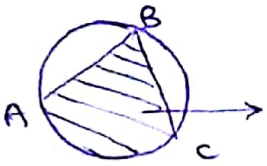


- Diameter - AB
- Radius - AC = BC = CD
- Chord - XY ; FG ; AB

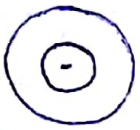
{AB is chord and also diameter, it is consided as a biggest chord in a circle}

- Tangent - $\overline{LM}$
- Point of Tangency - P
- Secant - EG
- Sector - ACD ; BCD ; ACB {ACB is also a Semi-circle and considered as a biggest ~~ch~~ sector}
- Segment - XYGF ; FGBA ; XYBA {Area enclosed b/t two chord}

ege



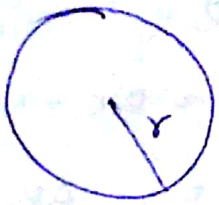
→ segment.

⇒  → concentric circles (or) Ring.

{Two circle have same center}

- Arc - A part of circle
- Circumference - Total length of an arc in a circle.
- Perimeter - sum of all sides.

- Area & Perimeter

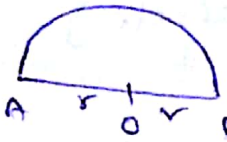
- 

Circle

$A = \pi r^2$

$P = 2\pi r$

$C = 2\pi r$

- 

Semicircle

$A = \frac{\pi r^2}{2}$

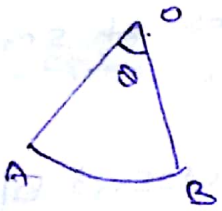
- $P = OA + \overset{\frown}{AB} + BO$

$P = r + \pi r + r$

$\boxed{P = \pi r + 2r}$

- $l = \overset{\frown}{AB} = \pi r$

$\boxed{l = \pi r}$

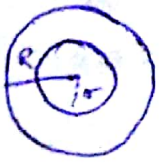
- 

Sector

$A = \frac{\theta}{360} \cdot \pi r^2$

$P = \frac{\theta}{360} \cdot 2\pi r + 2r$

$l = \frac{\theta}{360} \cdot 2\pi r$

- 

Ring

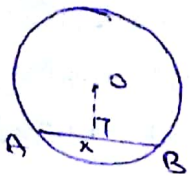
$A = \pi(R^2 - r^2)$

Properties

1. 

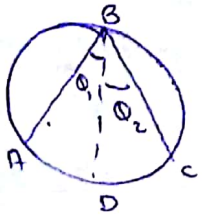
if two chord are equi-distance from center Then

$AB = CD$ if $h_1 = h_2$

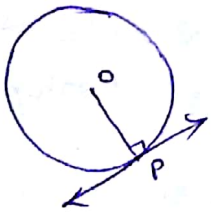
2. 

$AX = XB$ if

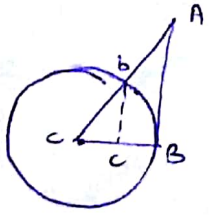
$\angle x = 90'$

3. 

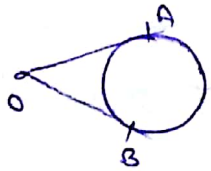
$\overset{\frown}{AD} = \overset{\frown}{DC}$ if $\theta_1 = \theta_2$

4. 

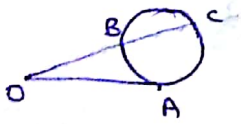
$\angle P = 90'$

5. 

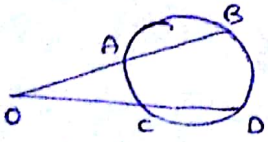
$DE = \frac{1}{2} \cdot AB$

6. 

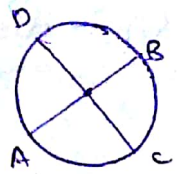
from external point only two tangent can draw.

7. 

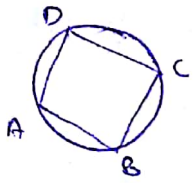
$OA^2 = OB \cdot OC$

8 

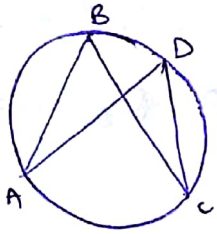
$OA \cdot OB = OC \cdot OD$.

9. 

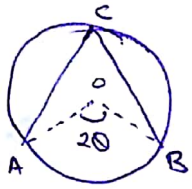
$OA \cdot OB = OC \cdot OD$

10. 

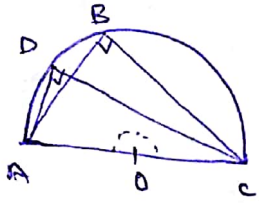
$\angle A + \angle C = \angle B + \angle D = 180^\circ$

11. 

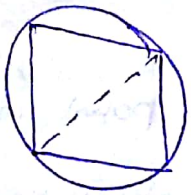
$\angle B = \angle D$, $\angle A = \angle C$

12. 

$\angle O = 2 \cdot \angle C$

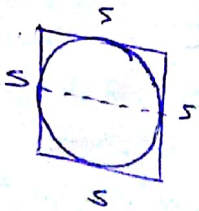
13. 

$\angle B = \angle D = 90^\circ$

14. 

$\sqrt{2} \cdot S = 2r$

$r = \frac{\sqrt{2} \cdot S}{2}$

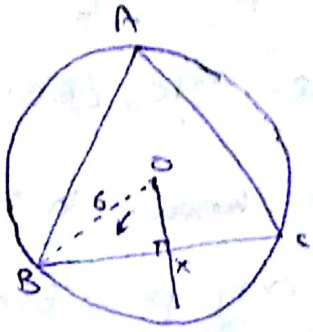
15. 

$2r = S$

$r = S/2$

Give

Qno1 cyclic Triangle, here triangle is equilateral Triangle whose all sides are equal and angle is 60° find area of ABC



sol · Given → radius = 6

$\angle ABC = 60$,

$\angle OBC = 30$, $\angle BxO = 90$, $\angle BOx = 60$

30 : 60 : 90

1 : $\sqrt{3}$: 2

3x(1 : $\sqrt{3}$: 2)

3 : $3\sqrt{3}$: 6

Why we are multiplying with three because already OB is given which is 6, is to get -6, which number is multiply & multiplied with -2 it is 3. so 3- is multiplied with all numbers.

so $\angle BOx = 60$ whose opposite side is Bx we required $Bx = 3\sqrt{3}$ //

As we know $BC = Bx + xC$

or

$BC = 2Bx$

$\boxed{BC = 6\sqrt{3}}$

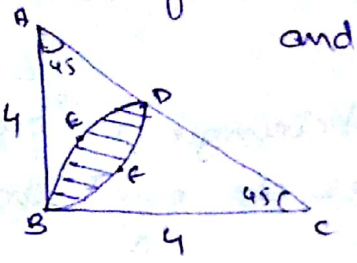
from area formula

$$A = \frac{\sqrt{3}}{4} \times (6\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 36 \times 3$$

$$= 27\sqrt{3} \text{ //}$$

Que-2 find area of shaded region in a triangle and it is a right angle triangle



Sol

$\angle ABC = 90, \angle BAC = 45, \angle BCA = 45$

We ~~know~~ In This triangle find ABED

it is only possible by ABC − CDEB

~~and~~ so $= \frac{1}{2} 4 \times 4 -$ area of sector

$= \frac{1}{2} \times 4 \times 4 - \frac{\theta}{360} \times \pi r^2$ $\{\theta = 45\}$

$= \frac{1}{2} \times 4 \times 4 - \frac{45}{360} \times \pi r^2$ $\{r = 4\}$

$= \frac{1}{2} \times 16 - \frac{45}{360} \times \pi \times 16$

area of 'ABED' $= 8 - 2\pi$

** here length of AB = length of BC = 4

So, area of ABED = area of CDFB

area of Δ^{le} ABC = area of + area of ABED + Area of CDFB
BEFD

we need

area of BEFD $= \frac{1}{2} \times 4 \times 4 - 2(8 - 2\pi)$

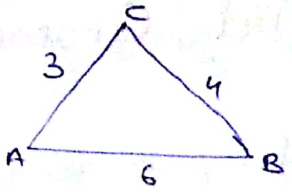
$= 8 - 2(8 - 2\pi)$

$= 8 - 16 + 4\pi$

$= 4\pi - 8$

Que-3 find area of triangle whose sides are 3, 6, 4 ? 6

sol let a = 6, b = 3, c = 4



$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

and $S = \frac{3+4+6}{2} = \frac{13}{2}$

$$= \sqrt{\frac{13}{2}\left(\frac{13}{2}-6\right)\left(\frac{13}{2}-3\right)\left(\frac{13}{2}-4\right)}$$

$$= \sqrt{\frac{13}{2}\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{5}{2}\right)}$$

$$= \sqrt{\frac{13 \times 35}{4 \times 4}}$$

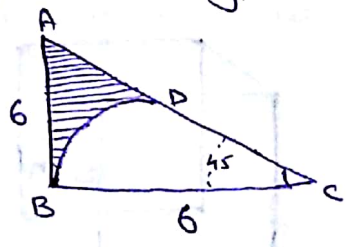
$$= \frac{1}{4}\sqrt{455}$$

$$= \frac{21.33}{4} = 5.33 //$$

Que-4 find area of shaded region in a triangle and it is a right angled triangle

Ans 4 BCD is a sector

$\angle BCD = 45°$, $\angle ABC = 90°$



area of ABD = Triangle - sector

$$= \frac{1}{2} \times 6 \times 6 - \frac{45}{360} \times \pi (6)^2$$

$$= \frac{1}{2} \times 6 \times 6 - \frac{45}{360} \times \pi \times 36$$

$$= 18 - \frac{9\pi}{2}$$

$$= \left(\frac{36 - 9\pi}{2}\right)$$

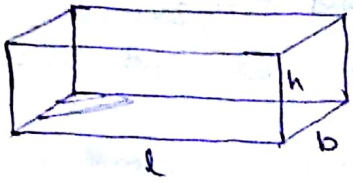
$$= \frac{7.7256}{2}$$

$$= 3.862 //$$

28/7/18

Solid Geometry

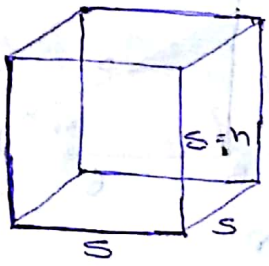
1. Cuboid



- Volume (V) = $l \times b \times h$
- Lateral surface area (L.S.A) = $2(l+b) \times h$
- Total Surface area (T.S.A) = $2lh + 2bh + 2lb$ = $2(lb + bh + hl)$
- Diagonal (D) = $\sqrt{l^2 + b^2 + h^2}$

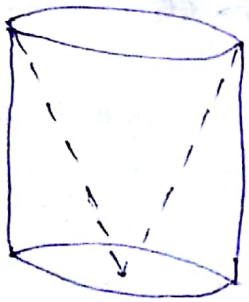
- L.S.A is area of all four wall
  - Front, back, Top and bottom
- T.S.A is area of all 6-sides.

2. Cube



- Volume (V) = area × height = $s^2 \times s = s^3$
- Lateral surface area = $4s^2$
- Total surface area = $6s^2$
- Diagonal (D) = $\sqrt{3}s$

3.

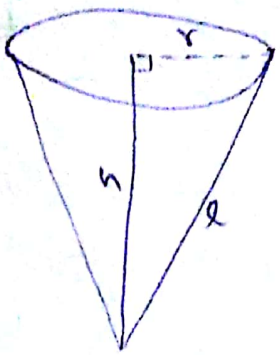


Cylinder

- Volume (V) = $\pi r^2 h$
- curved surface area (CSA) = $2\pi rh$
- Total surface area = $2\pi rh + 2\pi r^2$

Curved CSA = area of a cylinder {excluding the top and bottom circles}

4. CONE



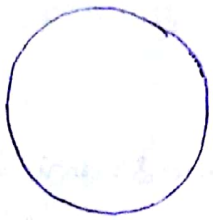
Volume (v) = $\frac{1}{3}\pi r^2 h$

curved surface area (CSA)
$= \pi r l$

$l = \sqrt{h^2 + r^2}$

Total surface area =
$= \pi r l + \pi r^2 + 0$
$= \pi r (r + l)$

5. Sphere



Volume (v) = $A \times h$
$= \frac{4}{3}\pi r^3$

C.S.A / L.C.S.A = $P \times h$

Total surface area = $4\pi r^2$

6 Hemi sphere

- Volume (v) = $\frac{2}{3}\pi r^3$ //

C.S.A = $2\pi r^2$ //

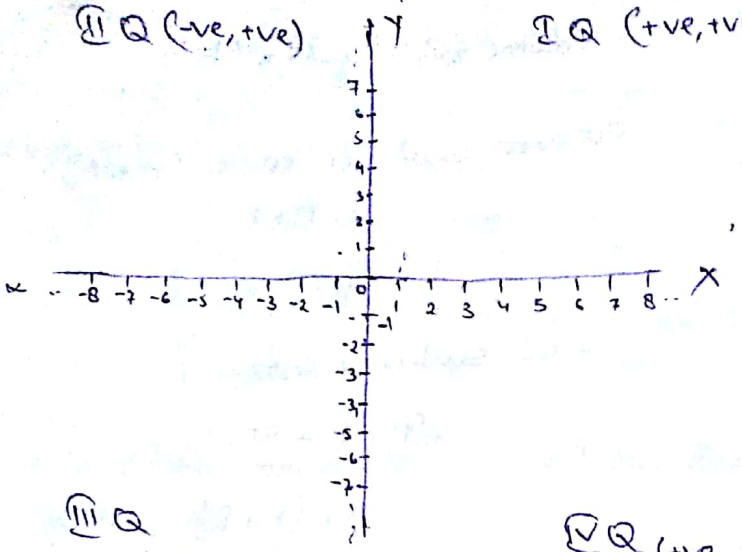
T.S.A = $2\pi r^2 + \pi r^2 + 0$

T.S.A = $3\pi r^2$ //

Co-ordinate Geometry.

II Q (-ve, +ve)

I Q (+ve, +ve)



III Q (-ve, -ve)

IV Q (+ve, -ve)

(x, y) → ordered pair.

→ if $2x - y < -6$ Then which of the following Quadrant is not satisfied by above inequation.

(A) I (B) II (C) III (D) IV (E) cannot be determine

Sol: answer - (d) IV

$$2x - y < 6$$

In IV-Qua $2(+ve) - (-ve) < -6$

$+ve < -6$

left side value will always be positive, so it is not satisfied in this eqn.

→ Point on x-axis $(x, 0)$

→ Point on y-axis $(0, y)$

→ line parallel to x-axis is $y = k$

→ " " " y-axis is $x = k$

→ Equation of x-axis is $y = 0$

→ " " y-axis is $x = 0$

→ line passing Through origin $y = mx$.

→ Slope of x-axis is $m = 0$

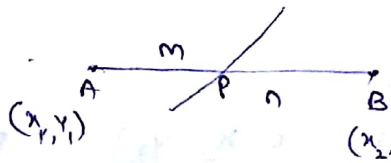
→ Slope of y-axis is $m = \infty$

- if $A(x_1, y_1)$ and $B(x_2, y_2)$ are two point Then distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

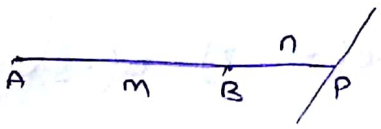
- Mid point of $AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Slope of AB $(m) = \frac{y_2 - y_1}{x_2 - x_1}$

➢ Line divides the line segment joining the points $A(x_1, y_1)$ and (x_2, y_2) in m and n find P.



$$P = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$

➢ if its ~~divide~~ cuts from outside of AB Then P = ?



$$P = \left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right]$$

→ Equation of line AB is $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$

→ A triangle have Three poin $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ Then its center point is

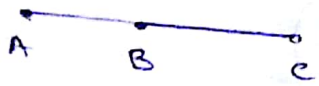
$$C_1 = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

→ Area of Δ^{le} ABC $= \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

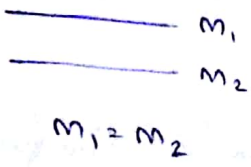
or

$$= \frac{1}{2}\begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix} \quad ; \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

→ If A, B, C are collinear Then



{ Area of Δ^{le} = zero
Slope of AB = Slope of BC



$m_1 = m_2$

Perpendicular

$m_1 \cdot m_2 = -1$

→ In Quadrilateral.

A(x_1, y_1) B(x_2, y_2) C(x_3, y_3) D(x_4, y_4)

~~Suppose~~ A(: Area of ABCD $= \frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_4 & y_2-y_4 \end{vmatrix}$

eg: A(2, 4) B(4, -3) C(-3, -2) D(-3, 4)

Area of ABCD $= \frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_4 & y_2-y_4 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 2+3 & 4+2 \\ 4+3 & -3-4 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 5 & 6 \\ 7 & -7 \end{vmatrix}$ (① ↘ 6, ② ↗ 7)

$= \frac{1}{2}\left|-35-42\right|$

$= \frac{1}{2}\left|-77\right| = 77/2$

Equations of lines

if $ax+by+c=0$ Then slope $m = -\frac{a}{b}$

$y = mx+c$ slope intercept form

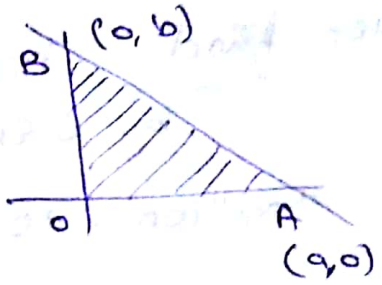
$y-y_1 = m(x-y_1)$ slope-point form

$\frac{x}{a} + \frac{y}{b} = 1$ intercept form

X-intercept $= a = OA$

Y-intercept $= b = OB$

Area of $\Delta^{le} = \frac{1}{2}|ab|$



31/07/18

Problem solved from ETS book

01/08/18

Counting

⇒ No. of items = (Max - Min) + 1

⇒ No. of Items in between = (Max - Min) - 1

Example: find, how many numbers are divisible by 6 & 8 from 1 to 1million.

Sol: 1million = 10,00,000

6 & 8 divides 24, 48, 72, 96...

So, to find out to numbers divisible by 6 & 8 Then divide 1million with 24

$\frac{10,00,000}{24} = 41,666.666$

= 41,666 //

{ consider only 41,666 not after '.' because if we values after '.' Then we will get more number.

Ex: how many numbers are divisible by 2 & 3 from 400 to 5000

Sol:

2 & 3 divides, ? 6, 12 ....

but if we consider from base or starting number which is divisible by 2 & 3 is '6'

now divide 400 with 6

$400/6 = 66.666$

= 67

{ consider it as 67 because we have to find values more Then 400. so it is 67.

now $\frac{5000}{6} = 833.33$

$= 833$

now consider only 833 dont add 1. we need numbers below 5000 divisible by 6.

402 4998

67 833

$67 \times 6 = 402$

$833 \times 6 = 4998$

No of items = $(833 - 67) + 1 = 767$.

COUNTING RULES

Sum (OR) $m+n$

Product (AND) $m \times n$

Que. In a class room There are 10-Girls and 20-boys In how many way a teacher can give a leader To the class

Sol: 10-Girls 20-boys

$10 + 20 = 30$ //

Que A house is having 4 main doors and 8 windows In how many ways a Thief can enter into The house

Sol: 4-main door 8-window

$4 + 8 = 12$

Que In how many ways a driver can pickup Mr Tom cruise from The school which contains 12-gates a drivers use a any of The gate atmost one.

Sol In any gate among 12.
↓ after entered in school he used
12 one gate remaing left gate are 11.

In and out
12 × 11 = 132

atmost one
- only one ~~one~~ or one

atleast one
one or more than one.

Que Find Total numbers which is divisible by 5 and ~~There is no repetition of the digits~~.

Sol for 5 (divisor) Then once-place should be 5 or 0

Three digit number $\underset{\downarrow}{9} \times \underset{\downarrow}{10} \underline{\quad} \; 0 = 90$

(or) $\underset{\downarrow}{9} \times \underset{\downarrow}{10} \underline{\quad} \; 5 = 90$

$+$

180

{ hundred Place never be '0'. So Total num is 9

Que Find total numbers which is divisible by 5 and There is no repetition of the digits.

Sol for 5 (divisor) Then once-place must be 5 or 0.

and no-repetition are allowed.

$\underset{\downarrow}{9} \times \underset{\downarrow}{8} \underline{\quad} \; 0 = 72$

$\underset{\downarrow}{8} \times \underset{\downarrow}{8} \underline{\quad} \; 5 = 64$

136

{ 8 numbers on 10th place because one number is 0 or 5 and second number is left side number which should not be repeated

eg 110 - X
100 - X
120 - ✓

Que In a town There are 4-letter boxes in how many ways Three letter can be posted in 4-letter boxes

Sol: $L_1 \times L_2 \times L_3$

$4 \times 4 \times 4 = 4^3 = 64$

-4

60

(-4 possibility of 3-letter in 1-boxes is 4 so it will be subtrad

Que An Indian Airlines offers Thalis which consist of 4-main course, 5-sweets, and 3-starters how many indian Thali can Airlines offers if it has

How many indian Thalis can be prepared if ~~every and~~ every Thali consist of starter, main course and sweets.

Sol

4-M, 5-S, 3-S

$4 \times 5 \times 3 = 60$ //

Que Exam consist of 3-Question with-4-options and 5 Question with (True or false) 2-options overall in how many ways 8 questions can be responded by the students

Sol

3 Q.M 5 T/F

4-opt 2-opt

$4^3 \times 2^5 = 64 \times 32 = 2048$

Que) 4-Question can be responded by the students which consists of a 5-options, in how many way four Question can be attempted which consist of each with 5-options in such a way that atleast one Question should be wrong

Sol:

Que1 Que2 Que3 Que4 (all have 5 options)

5^4 //

atleast one Que should be wrong

$5^4 - 1$

Que How many 4-digit number are there which are divisible by 6 using the digits

1, 2, 3, 4, 5, 6

Sol

6 × 6

1 2

1 6

2 4

3 2

4 4

5 2

56

64

(1 2 3 4 5 6) = 6

= 6 × 6 × 9

= 324 //

02/08/2018.

Permutation

Permutation mean Arrangement

⇒ $n P_r = \frac{n!}{(n-r)!}$ $\quad r \leq n$

⇒ $n P_0 = 1$; $n P_n = n!$

example: find $^{11}P_2$ and $^8P_3$

(a) $^{11}P_2 = 11 \times 10 = 110$

(b) $^8P_3 = 8 \times 7 \times 6 =$

$$^{11}P_2 = \frac{11!}{(11-2)!} = \frac{11!}{9!} = \frac{11 \times 10 \times 9!}{9!} = 110$$

Que ~~find~~ ~~$^7P_4$~~ =

⇒ In a bus, 7 seats available and 4 person has to occupy it.

Sol $^7P_4 = 7 \times 6 \times 5 \times 4$

$= 42 \times 20$

$= 840$

Arrangement

Row	circular
$n!$	$(n-1)!$

Que> 5-person gone for a movie, how many way in that They can arrange Themselves.

Sol $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

⇒ 5 persons so that two should sit beside each other.

Sol

$\Rightarrow 4! \times 2!$

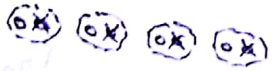
⇒ 4 couples went for a Movie how many ways They can arrange.

Sol

4 - couples → 8-member → 8! // { As there is no condition }

⇒ 4 couples but they should sit beside each other.

Sol



$= 4! \times 2! \times 2! \times 2! \times 2!$

⇒ 4 couples all mens on one side and womens on ~~on~~ one side.

Sol:

$\Rightarrow 2!\ 4!\ 4!$

⇒ 4 boys and 4 Girls should be arrange so That They should be alternate.

Sol:

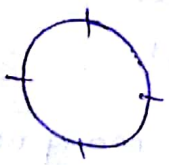
B – B – B – B – ⇒ $4! \cdot 4!$

G – G – G – G – ⇒ $4! \cdot 4!$ +

$2 \cdot 4! \cdot 4!$

⇒ 4 friends on dinner arrangement on a circular table.

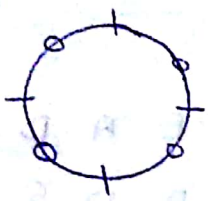
Sol:



$(4-1)! = 3!$

⇒ 4-boys and 4-girls went for a dinner so that they should sit alternate to each other.

Sol:



$(4-1)!\ (4\times3\times2\times1)$

$3!\times4!$

⇒ BIHAR – 5! (Possible words by using Alphabets)

⇒ Vowels together in BIHAR

Sol: (IA) B H R – 4! 2!

1 2 3 4

⇒ when BIHAR alphabets starts with consonants

Sol: 'or' B – – – – = 4!

'or' H – – – – = 4!

R – – – – = 4!

$3\times4!$

⇒ In how many ways APPLE ~~was~~ can arrange?

Sol APPLE = $\frac{5!}{2!}$ {2! because 'P' are repeated 2-times

⇒ In how many ways BANANA can arrange?

Sol BANANA = $\frac{6!}{3!\,2!}$ {3-'A', 2-'N'

⇒ CO-ORDINATION is word so that vowels should be To-gether.

Sol (O O I A I O) ← Vowels C R D N T N

1 2 3 4 5 6 7

$= \frac{7!}{2!}\times\frac{6!}{3!\,2!}$

⇒ In a function 4-guests should attend the gathering so that they all should sit in a wrong Arrangement.

Sol: Suppose - D L E A

$$\rightarrow {}^{n}P_4\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots\pm\frac{1}{4!}\right]$$

$${}^{4}P_4\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]$$

$$24\left[1-\frac{1}{1!}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}\right]$$

$$12-4+1=9 \;//$$

$9 =$

L	E	A	D
L	A	D	E
L	D	A	E
E	D	A	L
E	A	L	D
E	A	D	L
A	E	D	L
A	D	L	E
A	E	L	D

WRONG PERMUTATIONS:

$${}^{n}P_n\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots\pm\frac{1}{4!}\right]$$

COMBINATION - (selection)

$$nC_r = \frac{n!}{(n-r)!\,r!} = \frac{nP_r}{r!} \; ; \; r \le n$$

$$nC_0 = nC_n = 1 \; ; \quad nC_r = nC_{n-r}$$

eg: find $7C_3$, $15C_3$, $10C_2$?

(a) $7C_3 = \frac{7 \times 6 \times 5}{3!} = 35$

(b) $15C_3 = \frac{15 \times 14 \times 13}{3!} =$

(c) $10C_2 = \frac{10 \times 9}{2 \times 1} = 45$

⇒ There are four roses and 5-lilies how many ways They can arranges so That 6 should form bouquet.

Sol: $9C_6$

⇒ It should be arrange such that atleast 2-roses

$$\binom{R}{2}\binom{L}{4} + \binom{R}{3}\binom{L}{3} + \binom{R}{4}\binom{L}{2}$$

$$4C_2 \times 5C_4 + 4C_3 \times 5C_3 + 4C_4 \times 5C_2$$

$$\frac{24 \times 3}{2} \times \frac{5}{1} + \frac{4}{1} \times \frac{5 \times 4^2}{2 \times 1} + 1 \times \frac{5 \times 4^2}{2 \times 1}$$

$$= 30 + 40 + 10$$

$$= 80$$

⇒ There are 5-men and 3-women, no two women sit together.

Sol: $\overset{w}{\_}M\_M\_M\_M\overset{w}{\_}M\overset{w}{\_}$

$${}^6C_3 \times 3! \times {}^5C_5 \times 5!$$

⇒ BIHAR no two vowels to-gether

Sol: $\_B\_H\_R\_$

$${}^4C_2 \times 2! \times {}^3C_3 \times 3!$$

$$\frac{4\times3}{2} \times 2 \times 1 \times 6$$

$$12 \times 6 = 72$$

'or'

∆ vowels together

(A) BHR $- 4!\,2! = 48$

1 2 3 4

BIHAR $- 5! = 120$

now $120 - 48 = 72$

⇒ ~~Sele~~ SELECTION no two vowels together

Sol: $\_S\_L\_C\_T\_N\_$

$$= {}^6C_4\left(\frac{4!}{2!}\right) \times {}^5C_5 \times 5! = \frac{6\times5\times\cancel{4!}}{\cancel{4!}\,2!} \times \frac{4!}{2!} \times 1 \times 120$$

$$= \frac{6\times5\times4!}{2!\times2!} \times 120$$

$$= 90 \times 120 \times 2$$

$$= 10800 \times 2$$

$$= 21600$$

(WRONG M)

$$\left[\frac{9!}{2!}\right] - 6! \times \left(\frac{4!}{2!}\right) = \frac{9\times8\times7\times6!}{2!} - 6!\left(\frac{4\times3\times2!}{2!}\right)$$

$$= 252 \times 6! - 12 \times 6!$$

$$= 240 \times 720$$

⇒ 10 couple (10-men and 10 women) Possibility of shaking hand

Sol: $^{20}C_2$.

→ Between men and women

Sol: $^{10}C_1 \times {}^{10}C_1 = 100$

→ Between couple only

Sol: $^{10}C_1 \times {}^{1}C_1 = 10$

→ who are not spouse of each other

Sol: $^{10}C_1 \times {}^{9}C_1 = 90$

→ Between unmarried

$^{20}C_2 - 10$

$\frac{20\times19}{2\times1} - 10 \Rightarrow 190 - 10$

$= 180$

(or)

M×W	M×M	W×W
$^{10}C_1 \times {}^{9}C_1$	$^{10}C_2$	$^{10}C_2$
90	45	45

$= 180$

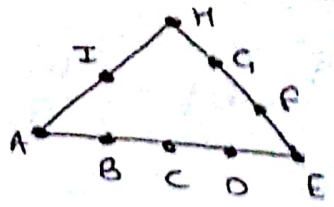
Que There are 60-boys and 40 girls ~~are~~ ~~There~~ among which 30 are siblings to each other. How many ways a boy and girl can be selected who are sibling to each other.

Sol: 30- siblings

30-B	30-G

$^{30}C_1 \times {}^{1}C_1 = 30$,

Que How many no. of Triangles can be drawn using the above points



Sol:

$$^9C_3 - {}^5C_3 - {}^4C_3 - {}^3C_3$$

→ 5 points are there on line AE and 3 points required for Δle

→ 4 points are there on line HE and 3 points required for Δle

→

PROBABILITY

04/08/18

$$P(E) = \frac{n(E)}{n(S)}$$

$n(E)$ = Event set ; $n(S)$ - sample set

$0 \leq P(E) \leq 1$; $P(\bar{E}) = (1 - P(E))$

* Probability on counting

Que: Tickets "1-100" probab.. of getting a number which is perfect squares

Sol: $n(E) = 10$; $n(S) = 100$

$$P(E) = \frac{10}{100} = \frac{1}{10}$$

Que: Probability of getting 3-digit no. which is divisible by 5 from 100-999.

Sol:

$\overset{9}{\downarrow}\ \overset{10}{\downarrow}\ 0 = 90$

$\downarrow\ \downarrow\ 5 = 90$ +

180

$= (999-100)+1$

$= 900$

$$P(E) = \frac{n(E)}{n(S)} = \frac{180}{900} = \frac{1}{5}$$

Que Probability of getting a no. which is divisible by 5 and 11 from 200 no's

Sol: 200 {55, 110, 165}

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{200}$$

→ Probability on Permutations

BIHAR – vowels comes together.

(1A) BHR – $4!\,2! = n(E)$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4!\,2!}{5!} = \frac{2!}{5} = \frac{2}{5}$$

Due 4-boys and 4-Girls went for a movie probability of Them sitting alternate to each other

Sol $B-B-B-B- = 4! \times 4!$

$G-G-G-G- = 4! \times 4!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2 \times 4! \times 4!}{8!} = \frac{2 \times 4!}{8 \times 7 \times 6 \times 5} = \frac{1}{35}$$

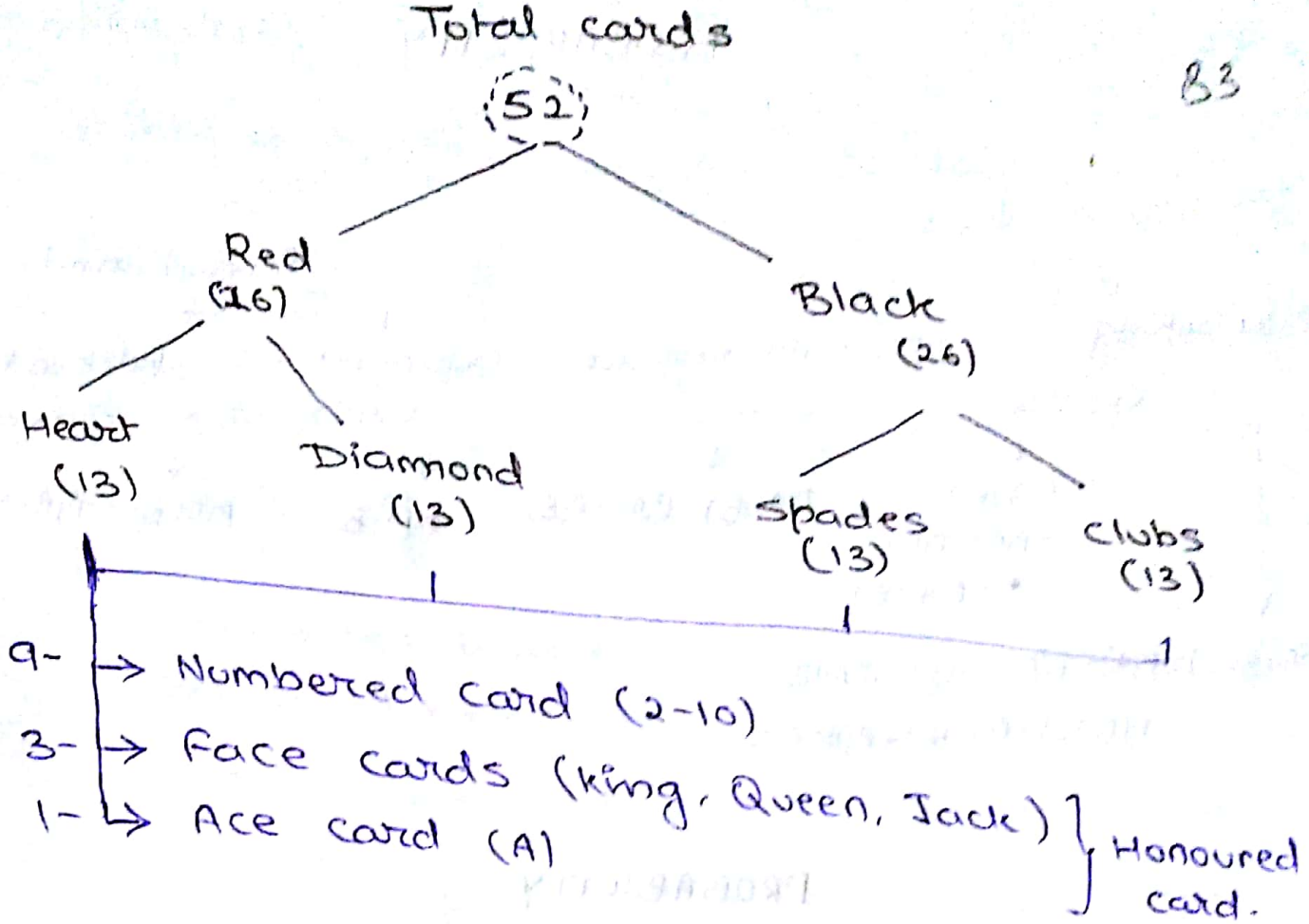
* PROBABILITY ON COMBINATION

→ 5 couples probability of shaking hand with only Their spouses

$$P(E) = \frac{n(E)}{n(S)} = \frac{5C_1 \times 1C_1}{10C_2} = \frac{5 \times 1}{45} = \frac{1}{9}$$

→ 6-Apples, 4 oranges probability of getting Two same fruit

Sol $$P(E) = \frac{n(E)}{n(S)} = \frac{6C_2 + 4C_2}{10C_2}$$



→ Probability on cards

(a) probability of one black card.

$$P(B) = \frac{26}{52} = \frac{1}{2}$$

(b) Probability of 3 black card.

$$P(B) = \frac{26C_3}{52C_3}$$

Que Probability of black or face cards

Sol $P(B \text{ or } F) = \frac{26+6}{52} = \frac{32}{52}$

Que Probability of king $P(K) = \frac{4}{52} = \frac{1}{13}$

Que Probability of Black & king ⇒ $P(B \cap K) = \frac{2}{52} = \frac{1}{26}$

PROBABILITY

- Sum-(or)-∪
  - Overlapping events → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - Mutually exclusive events → $P(A \cup B) = P(A) + P(B)$
- Product-and-∩
  - Dependent events → $P(A/B)$
  - Independents events → $P(A \cap B) = P(A) \times P(B)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

PROBABILITY

- All at a time
- one after the other (or) Success
  - With replacement
  - Without Replacement

Que Probability of selecting 1-face cards, 2-number card?

Sol: $$\frac{36C_2 \times 12C_1}{52C_3}$$

(a) With replacement = $\frac{36}{52} \times \frac{36}{52} \times \frac{12}{52}$

(b) Without replacement = $\frac{36}{52} \times \frac{35}{51} \times \frac{12}{50}$

Que Probability of selecting King, Queen & Jack

Sol: $$\frac{4C_1 \times 4C_1 \times 4C_1}{52C_3}$$

(a) With replacement = (i.e selecting 1 @ a time and replaced into total cards)

$$\frac{4}{52} \times \frac{4}{52} \times \frac{4}{52}$$

(b) Without replacement (i.e selecting 1 @ a time and throw it)

$$\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$$

PROBABILITIES ON DIES:

⇒ Probability of outcome by rolling a die which is factor of 2.

Sol $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$ {Factor of 2 = 1, 2}

⇒ Two dies rolled probability of getting same outcome

Sol $P(E) = \frac{6}{36} = \frac{1}{6}$

Que Two dies are rolled probability of getting a sum of output, which should be greater or Equal. to 10.

sol ~~Six have same output~~ ~~(5,5)~~ (5,5), (5,6), (6,5)

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$ (4,6), (6,4), (6,6)

Que Two dies are Rolled so That Probability of getting Value of 1st die should greater Than 2nd die.

Sol

1st die > 2nd die

15 //

15 down of diagonal
15 above of diagonal or (Principle ax)

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3)
(3,1) (3,2) (3,3)
(4,1) (4,2) (4,3) (4,4)
(5,1) (5,2) (5,3) (5,4) (5,5)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

~~dia~~ 6- diagonal.

Que: PROBABILITY ON SET THEORY.

⇒ Probability of hitting a target by soldier 1 is ½ Soldier 2 is ⅓, by soldier 3 is ¼. if all the Soldiers aim the target what is the probability That it will hit.

Sol

	S_1	S_2	S_3
$P(H)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$P(\bar{H})$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

$P(\bar{S}_1 \cap \bar{S}_2 \cap \bar{S}_3)$

$= P(\bar{S}_1) \times P(\bar{S}_2) \times P(\bar{S}_3)$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4} = 1 - \frac{1}{4} = \frac{3}{4}$ //

Que2 what is the probability That any one soldier will hit the target.

Sol $P(S_1 \cap \bar{S}_2 \cap \bar{S}_3) + P(\bar{S}_1 \cap S_2 \cap \bar{S}_2) + P(\bar{S}_1 \cap \bar{S}_2 \cap S_3)$

$= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right)$

$= \left(\frac{6}{24} + \frac{3}{24} + \frac{2}{24}\right) = 11/24$

one soldier will hit the target = $11/24$

Que3 what is the probability that any Two soldiers will hit the target.

Sol: $P(S_1 \cap S_2 \cap \bar{S}_3) + P(S_1 \cap \bar{S}_2 \cap S_3) + P(\bar{S}_1 \cap S_2 \cap S_3)$

$\left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}\right)$

$\frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{6}{24}$

Que4 what is the probability that ~~any~~ all ~~Two~~ soldiers will not hit the target.

Sol: $P(\bar{S}_1 \cap \bar{S}_2 \cap \bar{S}_3) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$ //

Que5 Probability that all three will hit the target.

sol: $P(S_1 \cap S_2 \cap S_3) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$

Que1 Question-1 can also be solved by

one-soldier-hit + two-soldier-hit + All three

$$\frac{11}{24} + \frac{6}{24} + \frac{1}{24} = \left(\frac{18}{24}\right) = \frac{3}{4} \quad //$$

Que4 In a company Machines M_1, M_2, M_3 produces 20% 30% & 50% of the items (Total production) respectively. If machine M_1, M_2, M_3 produces defective items as 2% 3% & 4%. If an item is selected what is the probability that it will be defective produced by The Machine That has a highest production?

sol: Total no. of items produced. = 100

100 (items)

M_1	M_2	M_3
20	30	50
Defective 2%	3%	4%

= 2% of 20, 3% of 30, 4% of 50

= 0.4 , 0.9 , 2

Total Defective = 0.4 + 0.9 + 2

= 3.3

for highest production is M_3 so

$$= \frac{2}{3.3} = 0.60_{\text{R}}$$

Qve Probability of getting a rain in hyderabad is 1/4 for any day. what is the probability that monday & friday will be rain in hyderabad for the given that week.

Sol

$$^2C_2 \times \left(\frac{1}{4} \times \frac{1}{4}\right) \times {}^5C_5 \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)$$

$$= \frac{1}{16} \times \frac{243}{256 \times 4}$$

$$= 0.0148$$

{ 1/4 for rain
{ 3/4 for no-rain

Qve Any two days in a week

Sol 1 week = 7 days

So

$$^7C_2 \left(\frac{1}{4} \times \frac{1}{4}\right) \times {}^5C_5 \left(\frac{3}{4}\right)^5$$

$$\frac{42}{21} \times \left(\frac{1}{16}\right) \times 1 \times \left(\frac{3}{4}\right)^5$$

$$21 \times \frac{1}{16} \times 1 \times \frac{243}{1024}$$

$$= 0.3114$$

ALGEBRA

INDICES:

$(3x)^2$ < Base - $3x$, Power - 2

$9x^2$ = Base - x, Power - 2, Co-eff - 9

Law of Indices:

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(a^m)^{1/n} = a^{m/n}$
- $a^{1/n} = \sqrt[n]{a}$
- $a^{-x} = 1/a^x$

$a^0 = 1$

$(a \cdot b)^m = a^m \cdot b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$(a \pm b)^m \neq a^m \pm b^m$

$a^m = a^n \Rightarrow m = n$

$a^m > a^n \Rightarrow m > n$

$a^m > b^m \Rightarrow a > b$

$\Rightarrow 6^{10} - 6^8 = 5^a \times 6^b \times 7^c$; find $a+b+c = ?$

$6^8(6^2 - 1) = 5^a \times 6^b \times 7^c$

$6^8 (35) = 5^a \times 6^b + 7^c$

$5^1 \times 6^8 \times 7^1 = 5^a \times 6^b \times 7^c$

$\Rightarrow a + b + c = 1 + 8 + 1 = 10$ //

$\Rightarrow 22^{22}$, 222^{222}, $2^{2^{22}}$ Which is greater?

a b c

$(2^4 - 2^5)$ $(2^7 - 2^8)$ $2^{\wedge\wedge}$

$2^{2^{22}} = (2^2)^{22} \Rightarrow 2^{44}$

$\Rightarrow$ ~~22^22~~ - let consider last digit and power

a) $2^{22} = 2^{10} \times 2^{10} \times 2^2$

b) $2^{222} = 2^{100} \times 2^{100} \times 2^{10} \times 2^{10} \times 2^2$ ← (b is greater

c) $2^{44} = 2^{10} \times 2^{10} \times 2^{10} \times 2^{10} \times 2^2$

find which is greater

Quantity A	Quantity B
⇒ 30^{40}	40^{30} ?
⇒ 30^{40}	50^{70} ?

1) 30^{40} , 40^{30}

$= (30^4)^{10}$, $(40^3)^{10}$

$= (810000)^{10} > (64000)^{10}$ ∴ Quantity A is greater

2) 30^{40} , 50^{70}

$= (30^4)^{10}$, $(50^7)^{10}$

$= (810000)^{10} < (78125 \times 10^7)^{10}$ ∴ Quantity B is greater.

Que what is a largest prime factor of 'A'

$A = (2^{31} + 2^{32} + 2^{33} + 2^{34})$

Sol $A = 2^{31}(1 + 2^1 + 2^2 + 2^3)$

$A = 2^{31} \times 15$

∴ largest prime factor is 5,

Functions

Domain - x

Range - $F(x) = y$

$f(x) = 3x+2$

$g(x) = 2x-3$ find $\frac{f(4)+g(2)}{f(1)}$

Sol:

$= \frac{3(4)+2+2(2)-3}{3(1)+2}$

$= \frac{14+1}{5} = \frac{15}{5} = 3$

find $fog(5)$

Sol: $f(g(5)) = f(2(5)-3)$

$= f(7)$

$= (3\times 7)+2$

$= 23$

find $gof(x)$

Sol

$g(f(x)) = g(3x+2)$

$= 2(3x+2)-3$

$= 6x+1$

$f(x) = \sqrt{3x-2}$ Then find Domain

Sol $3x-2 \geq 0$ } consider +ve

$x \geq 2/3$; $x \in (2/3, \infty)$

Defined functions:

$x \$ y = \frac{x+y}{x-y}$

: Solve $4\$(3\$1)$

Sol $4\$\left(\frac{3+1}{3-1}\right) = 4\$2 = \frac{4+2}{4-2} = \frac{6}{2} = 3$

Que $x^* = 1-x$; $[(1-x)^*]^* = x$

Sol $(1-x)^* = 1-(1-x)$

$= 1-1+x$

$(1-x)^* = x$

$x^* = x$

$1-x = x$

$\Rightarrow 2x = 1$

$x = 1/2$

$= a \oplus b = \left(\frac{a.b-b}{b^2}\right)$

$3 \oplus 2 = \left(\frac{3\times2-2}{4}\right) = \frac{6-2}{4} = 1$

EQUATIONS

Expression = constant

* Calculation Phase

EQUATION

- Monomials
  - Single variable
    - $3x - 7 = 8$
    - $3x = 15$
    - $x = 5$
  - Double variable
    - $3x+y = 9$
    - $x-y = 3$
    - $4x = 12$
    - $x = 3$
    - $x-y = 3$
    - $3-y = 0 \Rightarrow y = 0$
- Binomials
  - Quadratic Eq^n
    - $ax^2+bx+c = 0$
    - (α, β) are the roots
    - $\alpha+\beta = \frac{-b}{a}$; $\alpha.\beta = \frac{c}{a}$
    - $\alpha.\beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
    - $3x^2 - 10x + 8 = 0$
    - $F(-6,-4)$
    - $R\left(\frac{6}{3}, \frac{4}{3}\right)$
    - $x = 2, 4/3$.

Properties of Inequations:

If $a<b$

$\Rightarrow a<x<b$

$x \in (a,b)$

$\Rightarrow b<x<a$

$x \in (-\alpha,a) \cup (b,\alpha)$

$\Rightarrow a \leq x \leq b$

$x \in [a,b]$

$\Rightarrow b \leq x \leq a$

$x \in (-\alpha,a] \cup [b,\alpha)$

If $a>b$

$\Rightarrow a>x>b$

$x \in (-\alpha,a) \cup (b,\alpha)$

$\Rightarrow b>x>a$

$x \in (a,b)$

$\Rightarrow a \geq x \geq b$

$x \in (-\alpha,a] \cup [b,\alpha)$

$\Rightarrow b \geq x \geq a$

$x \in [a,b]$

$\Rightarrow a<b$

$b>a$

$\Rightarrow -a<-b$

$a>b$

$\Rightarrow a<b$

$\frac{1}{a}>\frac{1}{b}$ if only if $a.b>0$

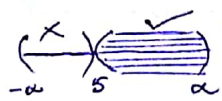
INEQUATIONS

- Monomials
  - Single variable
  - Double Variable
- Binomials
  - Quadratic equation

= $3x-7>8$

$3x>8+7$

$x>5$

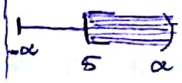


$x \in (5,\alpha)$

open interval

$3x-7 \geq 8$

$x \geq 5$



$x \in [5,\alpha]$

close interval

= $3x+y>9$

$x+y<-3$

= $3x+y>9$

$-x-y>3$

$2x>12$

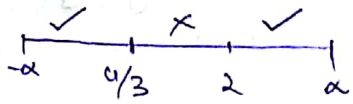
$x>6$

$x \in (6,\alpha)$

$3x^2-10x+8>0$

$F(-6,-4)$

$R\left(\frac{6}{3},\frac{4}{3}\right)$



$x \in (-\alpha,4/3) \cup (2,\alpha)$

or

$\frac{4}{3}>x>2$

or

$2<x<4/3$

$\Rightarrow 3x-7>8$ and $x-2<10$

$x>5$ and $x<12$

or $x \in (5,12)$

$5<x<12$

or

$12>x>5$

09/08/2018

FRAMING AN EXPRESSION

(+) - Add, Sum, Plus, more, greater, Increase, Increment, excess, In total after, Hence. etc.

(-) - Minus, Subtract, Difference less, lesser than, decrease, decrement, reduced, reduction, before, ago, dimnished, past etc.

(x) - Multiply, product, of, Numericals, times, etc.

(÷) - By, divides, divided by, distribute, ratio, to, etc

(=) - is equal to, as same as, ~~because~~ becomes, is, was, had, are, were, have, has, will, could, etc.

(≥) - Atleast

(≤) - Atmost

= Harry age is 10 more than twice Bens age

Sol: $H = 10 + 2B$ or $H = 2B + 10$

Qve 5 years ago Harry age was 3 times Ben's age

Sol: $(H-5) = 3(B-5)$

$H = 3B - 10$

Qve Harry age is 3-times Bens age

Sol $H = 3.B$

Qve 5-years Hence Marry age will be $3/5$ times of bens age.

Sol: $(M+5) = \frac{3}{5}(B+5)$

Qve Differenteeb/t Square of x and y is 25

$x^2 - y^2 = 25$

Qve Square of difference b/w x & y is 25

$(x-y)^2 = 25$

Que Y is 3 less than x?

$Y = x - 3$

Que If $\frac{1}{3}^{rd}$ of $\frac{1}{4}^{th}$ of a number is 15 Then $\frac{3}{10}^{th}$ of That number is?

$\frac{1}{3} \times \frac{1}{4} \times x = 15$

$x = 180$ //

$= \frac{3}{10}(x) = \frac{3}{10} \times 180 = 54$ //

Que 3-times The first of The 3 consecutive odd Integers is 3-more then twice The Third number.

Sol: Let three consecutive odd numbers are

$x, x+2, x+4$ $\Rightarrow$ $3x = 2x + 8 + 3$ $\{3x = 2(x+4)+3$

$\boxed{x = 11}$

$x+4 = ?$

$= 11+4 = 15$ //

Que Find a positive numbers which when increased by 17 is equal to 60 times The Reciprocal of The number?

Sol

$x + 17 = 60 \times \frac{1}{x}$

$x^2 + 17x - 60 = 0$

$F(20, -3)$

$R(-\frac{20}{1}, \frac{3}{1}) \Rightarrow x = 3$ //

Que Ten years ago Harry age is twice the Ben's age Ten years hence harry age will 3-times Ben's age, what is the present age of Ben?

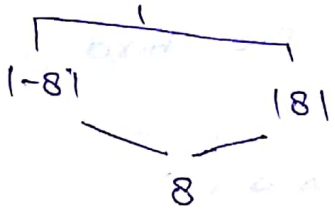
Sol:

$$H-10 = 2(B-10)$$
$$H+10 = 3(B+10)$$
$$\Rightarrow$$
$$H - 2B = -10$$
$$H - 3B = 20$$
$$B = -30$$

ABSOLUTE VALUES

$|x| = -8$ — Does not exist

$|x| = 8$



$= |3x-7| = 8$

$3x-7 = +8$ | $3x-7 = -8$

$3x = 15$ | $3x = -1$

$x = 5$ | $x = -1/3$

$= |3x-7| = |x-2|$

$3x-7 = +(x-2)$ | $3x-7 = -(x-2)$

$3x-x = -2+7$ | $3x-7 = -x+2$

$2x = 5$ | $4x = 2+7$

$x = 5/2$ | $x = 9/4$

$= |2x-7| < 9$

$-9 < 2x-7 < 9$

$-9+7 < 2x-7+7 < 9+9$

$-2 < 2x < 16$

$-1 < x < 8$

$x \in (-1, 8)$

$= |x-7| > 10$

$= -10 > x-7 > 10$

$-3 > x > 17$

$x \in (-\infty, -3) \cup (17, \infty)$

Set Theory:

* Set theory
* Collection of similar objects is known as a set.
* UNION: Means sum of each part of the figure is known as union.
* UNIVERSAL SET : Union + None

  None + None = μ

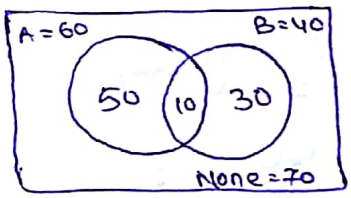
Formula.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

SETS

- overlapping sets
- Disjoints sets



60 students playing A-Games & 40 playing B-Games in a ground with 70 students who are not playing with 10 students are common to both games A & B.

μ = union + None

μ = (50 + 10 + 30) + None

= 90 + 70 = 160

Exactly one game by player = only 'A' + only 'B'

= 50 + 30

= 80

Exactly two game by player = 10

Atleast one game = 80 + 10 = 90 = Union = n(A∪B)

Not playing 'A' - game = 30 + 70 = 100

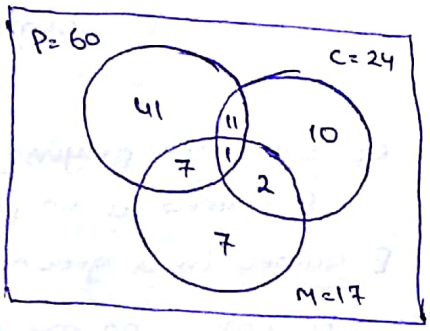
= 120 students is there in a class, students who attended Physics class are divisable by 2, Chemistry class divisible by 5 and Math divisible by 7 Then what is the value of None = ?

Sol: $U = 120$ $P = \frac{n}{2} - \{2, 4, 6 \ldots\ldots 120\} \rightarrow \frac{120}{2} = 60$

$C - \frac{n}{5} - \{5, 10, \ldots\ldots 120\} \rightarrow \frac{120}{5} = 24$

$M - \frac{n}{7} - \{7, 14, \ldots\ldots 120\} \rightarrow \frac{120}{7} = 17$

None = ?



$\frac{n}{2,5,7} = \{70, 140, 280 \ldots$

$(P \cap C) = \frac{n}{2,5} - \{10, 20, \ldots\ldots 120\}$

$= \frac{120}{10} = 12$

$(C \cap M) = \frac{n}{5,7} - \{35, 70 \ldots\}$

$(P \cap M) = \frac{n}{2,7} - \{14, 28 \ldots\}$

$= \frac{120}{14} = 8$

Union = 41 + 11 + 10 + 7 + 1 + 2 + 7

= 79

U = Union + None

120 = 79 + None

None = 120 − 79

= 41

⇒ Only Physics = 60 − (11+1+7) = 41

⇒ Exactly one subject = 41 + 10 + 7 = 58

⇒ Any two subjects = 7 + 11 + 2 = 20

⇒ Three subjects = 1

⇒ Atmost one subjects = one subject + None

= 58 + 41 = 99

⇒ Atleast two subjects = 20 + 1 = 21

⇒ Atleast one subjects = union = 79.

SEQUENCE

Sequence

- Irregular
  - eg: 2, 3, 5, 7.
- Regular
  - A.P
  - G.P
  - H.P

Que what is the 127^{th} decimal value of $\frac{2}{7}$?

Sol: $\frac{2}{7} = 0.285714285714285714\ldots$

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

(6, 6, 6 — repeating block of 6)

$6\overline{)126}$ → 21

126

127 → (1) Starting no.

127^{th} decimal value of exp $\frac{2}{7}$ is '2'

Que a, b, b, c, c, c, d, d, d, d ..... t_{233} = ?

t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9 t_{10}

Sol: $c = 3$, $\Sigma 3 = 1+2+3 = 6$; $\boxed{\Sigma n = \frac{n(n+1)}{2}} = \frac{3(3+1)}{2} = 6$

$d = 4$, $\Sigma 4 = \frac{4(4+1)}{2} = 2 \times 5 = 10 \rightarrow t_{10} \rightarrow d$

$10 - 4 = 6$

$= 6 + 1 = (7) \rightarrow t_7 \rightarrow d.$

- $T = 20$, $\Sigma 20 = \frac{20 \times 21}{2} = 210 \rightarrow t_{210} \rightarrow T$
- $V = 22$, $\Sigma 22 = \frac{22 \times 23}{2} = 253$

$(253 - 22) + 1 = 232$

*Required Answer = V

$= t_{n+1} = 3 \cdot t_n + 6$ (t_{n+1} term is thrice of the previous with adding 6-sum.

if $t_1 = 3$, $t_4 = ?$

$t_{1+1} = t_2 = 3 \cdot t_1 + 6$

$= 3 \times 3 + 6$

$= 15$

$t_3 = 3 \times t_2 + 6 = 3(15) + 6$

$t_3 = 45 + 6 = 51$

$t_4 = 3 \times t_3 + 6 \Rightarrow 3(51) + 6 \Rightarrow 153 + 6$

$t_{14} = 159$ //

Que t_{even} = constant ; t_{odd} = const

$t_3 = -3$ $t_6 = 6$; what is the value of sum of any two consecutive terms?

Soln: $t_1 = t_3 = t_5 \cdots = t_{odd} = -3$

$t_2 = t_4 = t_6 \cdots = t_{even} = 6$

$t_{10} + t_{10} = 6 - 3 = 3$; $t_9 + t_{10} = -3 + 6 = 3$

1- A.P

a_1 $a+d$ $a+2d$ $a+3d$

t_1 t_2 t_3 t_4

$t_n = a + (n-1)d$; $d = t_2 - t_1 = t_3 - t_2 = \cdots = t_n - t_{n-1}$

$S_n = \frac{n}{2}[2a + (n-1)d]$; $S_n = \frac{n}{2}[F + L]$; $d = \frac{t_n - t_m}{n - m}$

Que: 3, 8, 13, 18, 23 48

Sol: $t_{10} = a + 9d = 3 + 9(5) = 48$

$S_{10} = \frac{10}{2}(3 + 48) = 5 \times 51 = 255$

$S_{20} = \frac{20}{2}(2 \times 3 + 19 \times 5) = 10(6 + 95)$

$= 10(101) = 1010$

Qve $t_6 = 30$; $t_{12} = 120$; $t_{18} = ?$ in A.P.

Sol: $d = \frac{t_{12} - t_6}{12-6} = \frac{120-30}{6} = \frac{90}{6} = 15$

$t_6 = 30 \Rightarrow a + 5d = 30$

$a + 75 = 30$

$a = 45$

$t_{18} = a + 17d$

$= -45 + 17(15)$

$= 15 \times 14$

$= 210$

2. G.P

$a, ar, ar^2, ar^3 \ldots$

$t_1 \quad t_2 \quad t_3 \quad t_4$ $\qquad t_n = a.r^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$; $r<1$; $S_n = \frac{a(r^n-1)}{r-1}$; $r>1$

= 3, 6, 12, 24 ..... $t_5 = a.r^4 = 3(2)^4 = 48$

$S_6 = \frac{3(2^6-1)}{2-1} = \frac{3 \times 63}{1} = 189$

3. H.P

$H.P = \frac{1}{A.P}$

= 1, 1/3, 1/5, 1/7, --- H.P

Sol: 1, 3, 5, 7 .... A.P

$t_{10} = a + 9d = 1 + 9(2) = 19$ in A.P

$t_{10} = \frac{1}{19}$ in H.P

Que The amount in an account will doubles in every two years, After 10-years how much will be the amount it yield when he invest $300 in the beginning

Sol:

$x \xrightarrow{2} 2x \xrightarrow{4} 4x \xrightarrow{6} 8x \xrightarrow{8} 16x \xrightarrow{18} 32x$

$$S_6 = \frac{x(2^6-1)}{2-1} = \frac{x(63)}{1} = 300\,(63) = 18900$$

Que An Egyptian wants to form a pyramid using the cube blocks as 136 at the bottom and the rows will differ by 6 blocks as we keep on to the top. How many blocks can be used over all to form a pyramid.

Sol

$136, 130, 124 \cdots$

$t_1 \quad t_2 \quad t_3 \quad \cdots \quad d = -6$

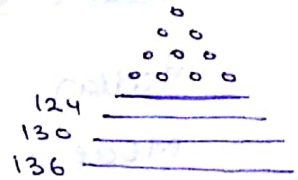
$t_n = 0$

$136 + (n-1) - 6 = 0$

$-6(n-1) = -136 \Rightarrow n-1 = \frac{136}{6}$

$n = 22.6 + 1$

$n = 23.6$



$t_{23} = a + 22d$

$= 136 + 22 \times (-6)$

$t_{23} = 136 - 132 \Rightarrow t_{23} = 4$ //

$$S_{23} = \frac{23}{2}\left(2 \times 136 + (23-1) - 6\right)$$

$$= \frac{23}{2}\left(272 - 132\right) = \frac{23}{2} \times 140$$

$= 23 \times 70 \Rightarrow 1610 + 6 = 1616$ //

STATISTIC

14/08/2018

Statistics

- Central Tendency
  - Range = Max − Min
  - Mean = A.M = Average = $\bar{x}$ = $\frac{\text{Sum of all items}}{\text{Total no of items}}$
  - Median = Middle most value
  - Mode = Most repeated item

$$\boxed{\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}}$$

- Deviation
  - QD
  - MD
  - SD
  - variance

Ex: 2, 3, 4, 4, 7

- Range = 7 − 2 = 5
- Mean = $\frac{20}{5} = 4$
- Median = 4
- Mode = 4

Symmetric Data

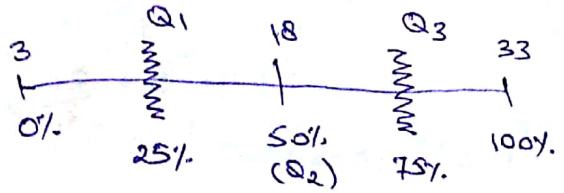
A.P $\overset{\checkmark}{\underset{\times}{\rightleftharpoons}}$ Symmetric Data

eg. 4, 8, 12, 16, 20

Mean = Median = Mode = 12.

Quartile Deviation $QD = \frac{Q_3 - Q_1}{2}$

$\{3, 8, 13, 18, 23, 28, 33\}$ $x = 7$



Q_1 = lower Quartile ; Q_3 = upper Quartile

n = Even = no of items	n = odd
$Q_1 = \left(\frac{n}{4}\right)^{th}$ observation	$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ observation
$Q_3 = 3\left(\frac{n}{4}\right)^{th}$ observation	$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ observation.

$Q_1 = \left(\frac{7+1}{4}\right)^{th}$ observation = 2^{nd} observation = 8

$Q_3 = 3\left(\frac{7+1}{4}\right)^{th}$ observation = 6^{th} observation = 28

$$QD = \frac{Q_3 - Q_1}{2}$$

$$= \frac{28-8}{2} = 10 //$$

Mean Deviation (M.D)

$$M.D = \frac{\left|\sum_{i=1}^{n} x_i - \bar{x}\right|}{n} \quad ; \bar{x} = \text{mean}$$

eg: $\{2, 3, 7, 8\}$ $\bar{x} = \frac{20}{4} = 5$

$$M.D = \frac{|2-5|+|3-5|+|7-5|+|8-5|}{4} = \frac{3+2+2+3}{4} = \frac{10}{4}$$

$M.D = 2.5$ //

Standard Deviation (σ)

$$\sigma = \sqrt{\frac{\left(\sum_{i=1}^{n} x_i - \bar{x}\right)^2}{n}}$$

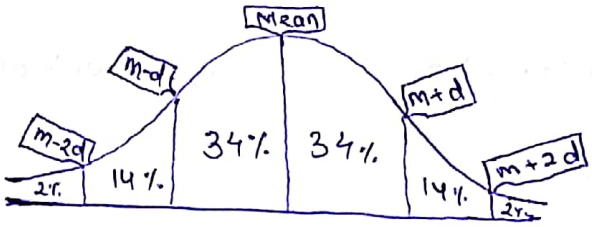
ex: {2, 3, 7, 8} $= \bar{x} = \frac{20}{4} = 5$

$$= \sqrt{\frac{(2-5)^2 + (3-5)^2 + (7-5)^2 + (8-5)^2}{4}}$$

$$= \frac{\sqrt{9+4+4+9}}{2} = \frac{\sqrt{26}}{2} = 2.54$$

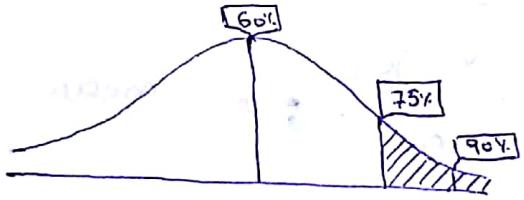
Variance $= \sigma^2$

Six-bell curve.



m = mean, d = S.D

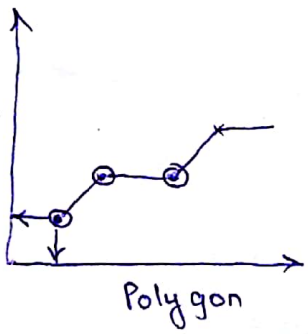
eg: mean - 60% SD = 15%.



m+d = 60+15
= 75%.

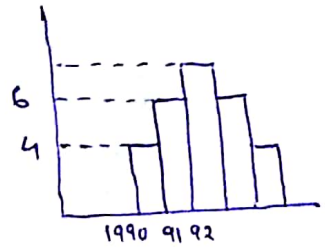
16% of 600 = 96 //

Polygon Graphs



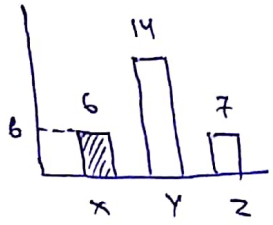
- Percentage
- Ratio
- Averages
- Fraction

Histogram



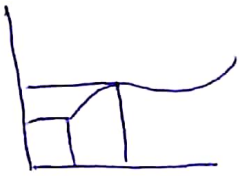
(1990-91)
(1991-92) $\frac{2}{4} \times 100 = 50\%$

Bar-charts

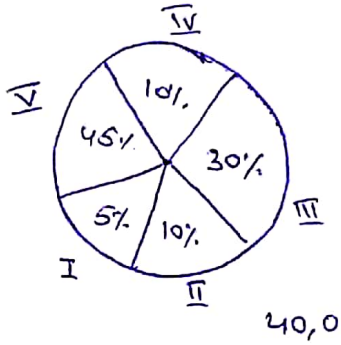


$\frac{6+14+7}{3} = \frac{27}{3} = 9$

Curve chart



Pie-chart



40,000

for - II

10% of 40,000 = ~~40,000~~

= 4000